이학박사학위논문

Prisoner's dilemma games on graphs

그래프 위에서 행해지는 죄수의 딜레마 게임

2013 년 2 월

서울대학교 대학원
물리·천문학부
윤창근

Prisoner's dilemma games on graphs

그래프 위에서 행해지는 죄수의 딜레마 게임

지도교수 강병남

이 논문을 이학박사 학위논문으로 제출함

2012년 11월

서울대학교 대학원

물리·천문학부

윤창근

윤창근의 박사 학위논문을 인준함

2012년 12월

위 원 장 ____________ (인)

부위원장 ____________ (인)

위 원 ____________ (인)

위 원 ____________ (인)

위 원 ____________ (인)

Prisoner's dilemma games on graphs

Chang-Keun Yun

Supervised by

Professor **Byungnam Kahng**

A dissertation submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy (Physics)

February 2013

Department of Physics and Astronomy
The Graduate School
Seoul National University

Abstract

Prisoner's dilemma games on graphs

Chang-Keun Yun
Department of Physics and Astronomy
The Graduate School
Seoul National University

Prisoner's dilemma(PD) game has been used widely in various disciplines as a tool to understand the mechanisms to evoke the cooperation although a player's favorable choice is not cooperative. Among a variety of explanations for the emergence of cooperation, the combination of evolutionary process and spatial structure is one of the successful hypotheses. In the first chapter, we review the spatial evolutionary PD games shortly. In the next two parts, we study the spatial evolutionary PD games in two detailed aspects.

In the second chapter, we study the PD games on several scale-free networks bridging between large-world and small-world types. Especially, we focus on the clusters of permanent cooperators. In small-world networks where the hubs are interconnected, one cooperator cluster is formed, and overall cooperation level is relatively high. On the other hand, in large-world networks where the hubs are separated, the clusters of cooperators with diverse sizes are formed, and the fraction of cooperators is not high. We investigate the cluster size distribution, changing networks from large-world ones to small-world ones, and find that the cluster size follows a power law at the transition point.

In the third chapter, we introduce mixed strategies into spatial evolutionary PD games. The probability of cooperation is used to represent the mixed strategies. As an application, we investigate the evolutionary stability in PD games with two mixed strategies on several types of regular graphs. A strategy which doesn't allow the invasion of other strategy is called an evolutionarily stable strategy. We find that under the deterministic game rules, there always exist evolutionarily stable strategies. These strategies can maintain the cooperation level against the invasion of other strategies. The introduction of mixed strategies in PD games can be the basis of more realistic PD games.

Keywords : prisoner's dilemma game, fractal network, large-world network, small-world network, mixed strategy, evolutionary stability
Student Number : 2004-20413

Contents

List of figures

List of tables

Chapter 1

Introduction to spatial evolutionary prisoner's dilemma games

1.1 Prisoner's dilemma

Cooperation among selfish individuals are universal in animal and human societies [1]. The cooperative behaviors are observed even in an RNA virus [2]. The origin of cooperation have been studied in various disciplines. The game theory is widely used as a theoretical framework to understand the emergence of cooperation.

A prisoner's dilemma(PD) game is a typical example of games describing the situations in which the interests of players conflict. A classical example of PD game is as follows. Prosecutors interrogate two suspects who are separated and remain silent. The prosecutors offer each of a suspect the same deal. If one betrays and testifies the other's crime while the other remains silent, the betrayer will go free and the one who remains silent will be sentenced to four years. If both remain silent, both will receive two year sentence. If both incriminate each other, they will get three-year sentence. The offer of the prosecutors can be summarized by the matrix presented in Table 1.

Prisoner's dilemma game can be generalized as shown in Table 2. When both players cooperate, they obtain the payoff R. If both defect, they receive the payoff P. If the choices of two players differ, the payoff of the cooperator is S and that of the defector is P. PD games satisfy the conditions, T $>$ R $>$ P $>$ S.

Table 1: The payoff matrix for a classical example of PD game. The values inside the table are the payoffs of suspect A.

	B stays silent	B betrays
A stays silent	–2	–4
B betrays	0	–3

Table 2: The payoff matrix for the general form of PD game. The values inside the table are the payoffs of player A.

	B cooperates	B defects
A cooperates	R	S
A defects	T	P

If rational, player A always choose defection regardless of the choice of player B. If player B cooperates, player A would better defect since T $>$ R. If player B defects, player A would better defect since P $>$ S. Player B also defects if rational. As a result, rational players defect, obtaining the payoffs P which is lower than R. In the previous classical example, the suspects will betray and incriminate the other if they are rational.

Many situations in real world can be viewed as PD games. One of the examples is the actions of nations to mitigate the global warming. To stabilize the concentration of CO_2 in the atmosphere, the emission of CO_2 should be reduced. Reducing the emission of the greenhouse gas, however, might lead to the decrease of national productivity. If a nation does not regulate the emission of CO_2, while the other nations do, the economic growth rate of this nation can be higher than those of the other nations.

Although the classical game theory predicts the defection of players, many examples of PD games in real world exhibit cooperation between players. To settle the disagreement, several mechanisms to support cooperation have been proposed: kin selection [3], group selection [4], direct reciprocity [5] [6], indirect reciprocity [7] [8], punishment [9] [10], rewards [11], voluntary participation [12] [13] [14].

Here, we introduce five mechanisms which Nowak summarized in [15].

Kin selection explains the emergence of cooperation from the perspective of a gene. Cooperation among genetic relatives is advantageous for the propagation of the gene. This mechanism can not explain the cooperative behaviors between the organisms which do not share genes.

Tit for tat is the symbolic phrase for direct reciprocity. The strategy that one cooperates only if the other cooperates is inferior to other strategies in a single game. In iterated games, however, this strategy can be superior [5].

Indirect reciprocity explains the altruistic behaviors among the individuals whose interactions are asymmetric and temporary. In these types of interactions, donors do not often receive the return benefits from the recipients. Instead, the altruistic behaviors of donors raise their reputations, which can increase the probability to receive help from others.

Group selection considers the competition between groups. The group of cooperators can have an an advantage in competition with that of defectors, even if an cooperator as a individual is at a disadvantage in competition with defectors. This can lead to the prosperity of cooperator groups and the die-off of defector groups.

Network reciprocity takes spatial or population structure into consideration. An individual interacts with some more often than with others. A network can be constructed from the interactions among individuals. Clusters of cooperators can be formed among the individuals with strong interactions. Network reciprocity is closely related to the spatial evolutionary games which is introduced in next section.

1.2 Spatial evolutionary prisoner's dilemma game

The pioneering works of Nowak and May [16] [17] introduced the spatial evolutionary PD games, which are the spatial versions of the evolutionary PD games. The evolutionary games [5] assume Darwinian selection instead of the rationality of players. Players have intrinsic strategies. In the evolutionary processes, the players with the successful strategies flourish and the ones with the unsuccessful strategies vanish. In the spatial versions, a player can only interact with the spatial neighbors. Nowak and May showed in the spatial evolutionary PD games, cooperators can survive, forming the clusters. The spatial evolutionary games attracted the statistical physicists' interests, for similarities between the spatial evolutionary games and the spin models. The strategies and payoffs of players are analogous to the states and interaction energies of spins.

Various mechanisms are suggested to enhance cooperation in the spatial evolutionary PD games. Some examples are as follows: reward mechanism [18] [19], preferential selection [20] [21] [22] [23] [24] [25] [26], master-follower asymmetry [27] [28], dynamical rules [29] [30], partner switching [31], social diversity [32], separation of interaction layer and learning layer [33] [34], interaction stochasticity [35].

The common features in most of the mechanisms are to increase the inhomogeneity among the players. In the preferential selection mechanism, the higher payoff of a player's payoff leads to the higher probability that the player is chosen as a reference by the neighbors is. The master-follower asymmetry mechanism assumes two types of players: one type of players accept the strategies of neighbors with high probability and the other type of players with low probability. The social diversity mechanism supposes the inhomogeneity of a player's influence, so-called diversity which is a scaling factor to map the payoff of a player to the fitness. The inhomogeneity among the players facilitates the forming of the cooperator clusters.

1.3 Spatial evolutionary prisoner's dilemma game on scale-free networks

Most of the early works studied the games on square lattice. With the advent of the concept of scale-free networks [36] [37], PD games on scale-free networks also have been studied. Scale-free networks are the networks with inhomogeneous degree distributions following the power-law, $P(k) \sim k^{-\alpha}$. Many real systems can be described as scale-free networks. Santos and Pacheco [38] [39] [40] claimed that cooperation can be enhanced in the games on random scale-free networks more than on some other types of graphs. The game rules suggested by Santos and Pacheco assume that the payoff of a player is the sum of the payoffs earned from the games with neighbors. Hence, in their rules, the influences of players on hubs are generally stronger than those on the other nodes. A defector on a hub expels cooperators in the neighborhood. As a result, the payoff of the player on the hub decreases. On the other hand, A cooperator on a hub fosters cooperators in the neighborhood. Therefore, the payoff of the cooperator on the hub increases. When hubs are connected, it is highly probable for defector hubs to change into cooperator hubs. As a result, a cluster of hub cooperators forms, spreading the strategy of cooperation to non-hubs. After, Devlin, *et al.* [41] exhibited the strong correlation between the standardized variance of the degree distribution and the density of cooperators in a variety of random networks including random scale-free networks.

Researchers studied the evolutionary PD games on structural variations of scale-free networks by the game rules proposed by Santos and Pacheco. Assenza, *et al.* [42] reported that cooperation is enhanced in the highly clustered scale-free networks. Pusch, *et al.* [43] showed the enhancement of cooperation in assortative scale-free networks for large temptation value. Chen, *et al.* [44] studied the evolutionary PD games on scale-free networks with community structure and concluded that the direct connections between hubs play more important role in enhancement of cooperation in these networks than the inhomogeneity of degree distribution.

However, several researches raised questions about the effect of scale-free network structure on the enhancement of cooperation. In the rules proposed by Santos and Pacheco, the payoff of a player is directly related to the number of neighbors. The introduction of the normalized payoffs [45] [46] [47] [48] and the participation costs [49] can weaken cooperation in the evolutionary PD games on scale-free networks. These mechanisms reduce the inhomogeneities of players' payoffs, leaving the structural inhomogeneities unchanged.

1.4 Rules for spatial evolutionary PD games

In this section we discuss the general rules for spatial evolutionary PD games.

1.4.1 Typical processes of games

In the spatial evolutionary games, each player has its own strategy, and plays games with the nearest neighbors. The payoff of a player is determined by the strategies of the player and the neighbors. In Darwinian process, the fitter can survive. The fitness of a player is calculated from the payoff. If the fitness of a player is lower than those of the neighbors, the player imitates the strategy of the player which has the higher fitness. In the terms of strategy, the worse strategy is replaced by the better one. The strategy imitation of players or the replacement of strategies results in the change in the local environment. This alters the fitnesses of players. These processes are iterated. These are the typical processes of the spatial evolutionary games.

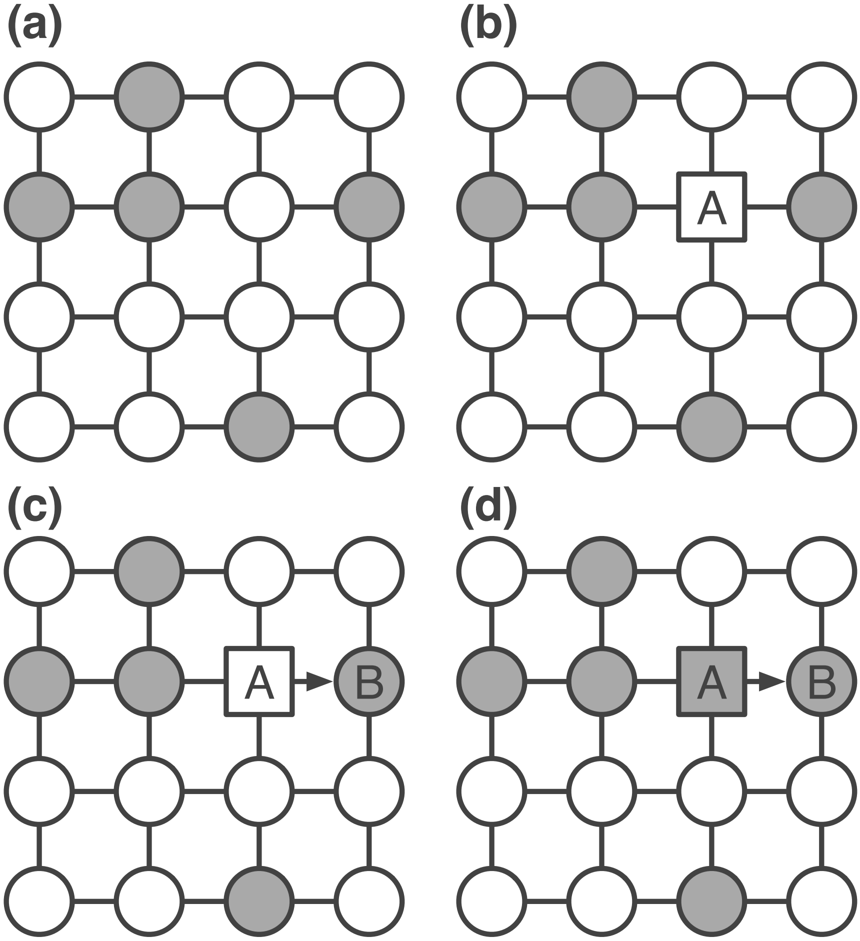


Figure 1: Processes of the spatial evolutionary PD game. (a) calculation of fitnesses, (b) selection of a candidate for the strategy update, (c) selection of a reference neighbor, (d) strategy update.

The typical processes are composed of the following steps; initialization, calculation of payoffs(fitnesses), selection of (a) candidate player(s), selection of a neighbor for a reference, strategy imitation. After initialization, the next steps are iterated. A candidate player means the player which tries to change its strategy. A reference player is a selected neighbor of the candidate, and if some conditions are satisfied, then the candidate imitates the strategy of the reference.

At the initialization step, the initial strategies of players are determined. In general cases, the strategy of cooperation is assigned to certain portion ρ_C^0 of players which are randomly selected.

Each step is conducted under the predefined rule which can be varied by the objective of research.

1.4.2 Payoff matrix

The canonical form of payoff matrix for prisoner's dilemma games is represented with four variables, T, R, P, S, as introduced in Table 2. Here, T $>$ R $>$ P $>$ S.

The canonical payoff matrix is rarely used in the the studies on the spatial evolutionary PD games. Instead, the simplified versions of payoff matrix are widely used. Here, we introduce two types of the simplified payoff matrix.

First type of the simplified payoff matrix introduced by Nowak and May [16] has the values T = b, R = 1, P = S =0, where $b > 1$. In this type of payoff matrix, the number of parameter is only one, and the parameter b is called as a temptation value.

Second type [50] has T = b, R = $b - c$, P = 0, S = $-c$, where $b > 0$, $c > 0$ and $b > c$. This type needs two variables. Some researches rescaled this type to have a single parameter. The rescaled payoffs are T = 1, R = $1 - r$, P = 0 and S = $-r$, where $r = c/b$.

1.4.3 Fitness

The payoff matrix determines the payoff of a player earned from the games with neighbors. A fitness of a player, a indicator of survivability is determined by the payoffs of the player. The higher fitness of a player raises the probability that the player survives and the strategy of the player spreads.

Many studies have used the accumulated payoff, i.e. the sum of the payoffs from the games with neighbors as a fitness of a player. Some researches employed the modified versions of fitness to control the inhomogeneity among players. For instance, Tang, *et al.* [51] used an average payoff $F_i = P_i/k_i$ as a fitness of player i, where P_i is an accumulated payoff of player i and k_i is the number of neighbors. In the work of Szolnoki, *et al.* [48], the fitness of player i is defined by $F_i = \alpha P_i + (1-\alpha)P_i/k_i$, where α is a normalization parameter. Jiang, *et al.* [52] used $F_i = P_i^{\alpha}$ as a fitness.

1.4.4 Synchronous update vs. asynchronous update

In the games with a synchronous update rule, all the players have the chances to change strategies at one time step. On the other hand, an asynchronous update rule allows only one player to change the strategies.

Huberman and Glance [53] introduced the asynchronous update rule. They criticized the synchronous update rule introduced firstly by Nowak and May [16], since the synchronous rule assumes the existence of a global clock, which is unrealistic. This dispute is still ongoing [54] [46], however has weakened with the introduction of stochasticity in other rules, especially transition probability. In the presence of stochasticity, the results of PD games updated synchronously become similar to those of PD games updated asynchronously. The stochasticity brings an effect to limit the number of players to change strategies.

1.4.5 Selection of candidate players for updating strategies

In the games with an asynchronous update rule, one selected player has a chance to change its strategy. In general cases, a candidate player is chosen randomly.

There are some exceptions. In BD(birth-death) update rule introduced by Ohtsuki, *et al.* [55] [56], a candidate player is selected with the probability proportional to fitness.

1.4.6 Selection of a neighbor for a reference

In evolutionary PD games, players try to imitate the strategies of players with better fitnesses. In most of the spatial evolutionary PD games, a player use one of the nearest neighbors as a reference. Nowak and May introduced a deterministic rule in their initial works [16] [57]. Under the deterministic rule, a player choose the neighbor with the highest fitness as a reference. Later, Nowak, *et al.* [58] [17] introduced the stochastic rule. The probability that player i choose player j as a reference among the neighbor players is proportional to ${f_j}^m$, where m is the parameter to control the stochasticity. In the limit of $m \to \infty$, this stochastic rule is equivalent to the deterministic rule. Many recent researches used a random selection rule, which is equal to $m = 0$ case. In the random selection rule, one neighbor is selected randomly with equal probability, regardless of fitness. Later, Wang and Perc [25] revived the idea of selecting the reference neighbor with the fitness-related probability.

1.4.7 Adoption probability

In the early studies of the evolutionary spatial PD games [16] [53], a player adopts the strategy of a reference neighbor only if the payoff of the player is lower than that of a reference neighbor. Szabó *et al.* [59] introduced the stochastic rule, in which player i adopts strategy of player j(reference neighbor) with probability

$$W(i \to j) = (1 + exp[-(f_j - f_i)/K])^{-1}. \tag{1.1}$$

Here, f_i and f_j are the fitnesses of player i and j, and K is the control parameter for noise level. This rule allows the irrational choices of players; it is possible for players to adopt the strategies of players with lower fitnesses. Hauert and Doebeli [60] introduced other type of stochastic rule. In their rule, player i adopts the strategy of player j with probability $W(i \to j) = (f_j - f_i)/D$ where D is $T - S$. Here, the fitness of a player is the average payoff. Their rule does not allow the irrational choices of players; players do not adopt the strategies of players with lower fitness. Santos and Pacheco [38] modified the stochastic rule introduced by Hauert and Doebeli to apply to PD games on the scale-free networks; player i adopts the strategy of player j with probability $W(i \to j) = (f_j - f_i)/(Dk_>)$ where $k_>$ is the largest between k_i and k_j, the numbers of neighbors of player i and j. Here, the fitness is the sum of payoffs from the games with neighbors. In Santos and Pacheco's rule, the probability that hubs change their strategies is relatively low.

Chapter 2

Prisoner's dilemma games on hierarchical model

2.1 Introduction

Nowak and May [16] introduced the prisoner's dilemma games on spatial structure, and showed the neighbor interaction on spatial structure can induce the formation of the cooperator clusters.

The spatial structure which Nowak and May used is square lattice. All players have the same number of neighbors in square lattice. In reality, however, the distributions of the number of neighbors are often inhomogeneous and follow a power law. These type of structures are called as scale-free networks [36] [37].

Santos and Pacheco [38] showed that the density of cooperation can be enhanced in small-world scale-free networks than in Euclidean space. The average distance l scales logarithmically in small-world scale-free networks. After Santos and Pacheco's study, variants of the evolutionary games on small-world scale-free networks have been studied.

Many scale-free networks in the real world are, however, not small worlds. These networks have modular structure, which is often hierarchically organized [61]. They have disassortativity, i.e, the strong repulsion between hubs. The average distance scales in a power-law manner [62] [63]. Such networks are called large-world or fractal networks. Social networks are often between small-world and large-world networks [64].

The connection between hubs is one of the main factors to enhance the level of cooperation in PD games on small-world scale-free networks. In large-world scale-free networks, however, the connections between hubs are extremely rare. Hence, the cooperation level on large-world scale-free networks can differ from that on small-world ones. The results of PD games on small-world scale-free networks therefore may not explain the characteristics of cooperation in some real world networks which are large-world.

We study the prisoner's dilemma game on model networks introduced in [65]. In this network model, a network can be transformed from large-world to small-world via long-range bond probability p. A percolation transition of cooperator clusters is found in the parameter space (p,b). The cluster size distribution follows a power law at the transition point. We also study prisoner's dilemma games on World-Wide Web(WWW), and the cluster size distribution near the percolation threshold follows a power law. The critical behavior results from the combined effects of stochastic processes in the PD game and the heterogeneity of complex network structure.

This study was published in [66].

2.2 Hierarchical network model

M. Hinczewski and A. N. Berker introduced a scale-free hierarchical network model [65] based on a hierarchical lattice introduced in [67].

2.2.1 Construction rule

In hierarchical network model, a network is constructed by iterating the procedure depicted in Fig. 2. The model assumes two type of links: expandable links(depicted

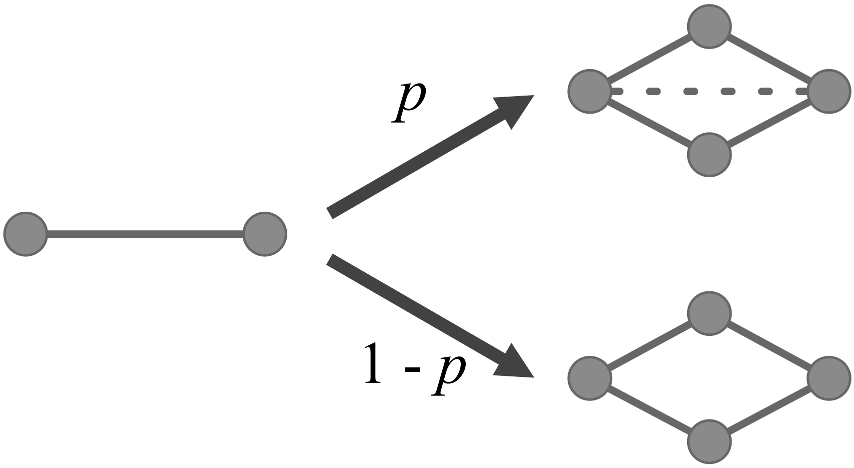


Figure 2: Construction of the hierarchical network.

as solid lines) and non-expandable links(depicted as dashed lines). Starting from a expandable link, in each step, every expandable link is replaced by the two parallel paths of two expandable links with probability $1-p$ or the two parallel paths of two expandable links and one path of one non-expandable link with probability p. This procedure is repeated until the desired network size is gained.

2.2.2 Network characteristics

Let us consider a network is constructed after the n construction step [65]. The number of nodes N is $\frac{2}{3}(2+4^n)$. The average degree $<k>$ is $3+p-\frac{3(2+p)}{2+4^n}$. In the limit of $x \to \infty$, $<k>$ is $3+p$. The network is scale-free and the degree exponent γ is 3.

When $p=0$, the average distance l is proportional to $N^{\frac{1}{2}}$, which means the network is large-world. When $p=1$, l scales logarithmically as $\ln N$, which means the network is small-world. The value of p controls the large-world property of the networks.

2.3 Rules for evolutionary prisoner's dilemma games

Payoff matrix determines a player's payoff earned from the game with other player. Here, we use the simplified version of payoff matrix for prisoner's dilemma game introduced in [16] and [60].

	C	D
C	1	0
D	b	0

First column is the strategy of player A and first row is the strategy of player B. Values in matrix are the payoffs of player A. For prisoner's dilemma game, b should be greater than 1.

We use the evolutionary prisoner's dilemma rules introduced in [38]. Details of the rules are as follows. Initially, each player is given one of the strategies, C(cooperator) or D(defector) randomly with equal probability. Once the strategies of the players are decided, the accumulated payoffs of the players can be calculated. In our study, the accumulated payoff $P_x$of player x is defined as the sum of the payoffs from the game with the neighbors. After the payoffs are determined, every player has the chance to imitate the strategy of the neighbor which has better payoff. Each player(player x) chooses one reference player(player y) among neighbors at random. If $P_x \geq P_y$, player x doesn't change its strategy. If $P_x < P_y$, player x imitate the strategy of player y with probability $(P_y - P_x)/(b \cdot max(k_x, k_y))$. Here, k_x and k_y) is the number of neighbors of player x and player y) respectively, and $max(k_x, k_y)$ is the largest between k_x and k_y. We repeat two processes of the imitation of strategies and the calculation of the accumulated payoffs for 2×10^4 steps.

The rule for updating a strategy does not allow the errors of players: players do not imitate the strategies of the neighbors which have lower payoffs.

2.4 Simulation results and discussions

2.4.1 Results on hierarchical networks

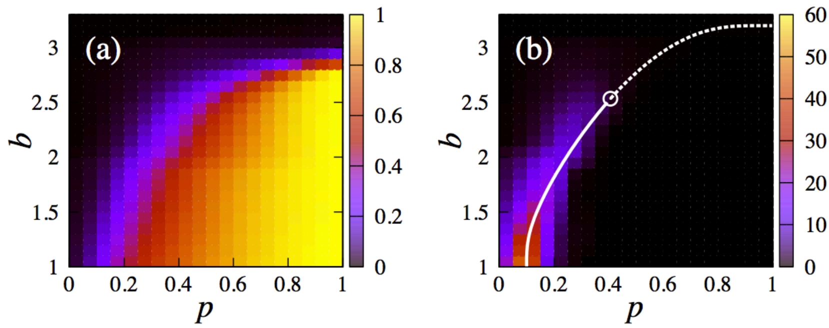


Figure 3: (a)Giant cluster size and (b)susceptibility in the hierarchical network with N = 10,924. The curves are the peak positions of susceptibility. The solid(dotted) curve represents continuous(discontinuous) percolation transition.

We simulate the evolutionary prisoner's dilemma games on hierarchical networks, varying p, b. The number of players(the number of nodes in a network) is 10,924. Data are averages over 100 network configurations. We focus our interests on permanent cooperators. A permanent cooperator is defined as the player which select strategy C for the last 10^4 steps. A permanent defector is defined similarly: the player which choose strategy D for the last 10^4 steps. The rests are unstable players, which change their strategy continuously.

Firstly, we investigate the giant cluster size of the permanent cooperators(Fig. 3(a)). For fixed b, the giant cluster size is larger for large p than for small p. For small b, the giant cluster size increases gradually with p; for large b, the giant cluster grows drastically near a certain p point. For fixed p, the giant cluster size is larger for small b than for large b. For small p, the giant cluster size decreases gradually with increasing b; for large p, the giant cluster size drops drastically near a certain b point. The giant cluster disappears beyond $b = 3.0$ regardless of b.

Secondly, we investigate susceptibility $\chi = \sum_s' s^2 n_s$. Here, $\sum'$ denotes the summation except the giant cluster, and n_s is the number of s-sized clusters divided by the system size. The higher diversity of clusters raises the value of susceptibility. The results are shown in Fig. 3(b). The curves in the figure are the loci along the peaks of susceptibility. The positions of peaks mark the phase boundary $p_c(b)$ across which the giant cluster grows to a macroscopic-scale cluster. The susceptibility has the maximum value near the point (0.1, 1.0). Along the curves, the susceptibility diminishes gradually until the tricritical-like point (p_t, b_t) represented as a circle. Beyond this point, the susceptibility seems to disappear. This suggests that for a fixed $b < b_t (b > b_t)$ or $p < p_t (p > p_t)$, the giant cluster of cooperators grows continuously(discontinuously). The estimated value of p_t is about 0.4. This is similar to $p^* \simeq 0.494$, which is the boundary between the large-world and the small-world networks as determined on the basis of the thermal transition patterns of the Ising model [65].

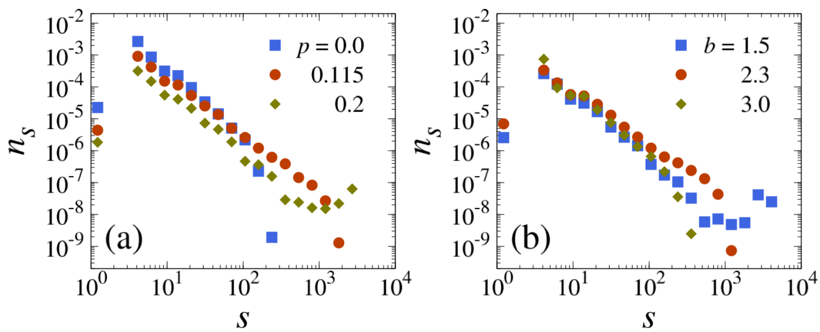


Figure 4: Size distribution of cooperator clusters in the hierarchical network with N=10,924. (a)$b = 1.7$, (b)$p = 0.15$

Next, we investigate the cluster-size distribution n_s near the continuous percolation transition threshold(Fig. 4). Data are averages over 500 configurations. Fig. 4(a) reveals n_s against s for $b = 1.7$ and several values of p. For the large-world network($p < p_c$), $n_s(p)$ shows a subcritical behavior; n_s follows a power law for small s, and n_s decays exponentially for large s. At $p_c \cong 0.1$, n_s follow a power law $n_s \sim s^{-\tau}$ with $\tau \approx 1.85 \pm 0.1$. For $p > p_c$, n_s shows a supercritical behavior. The similar behaviors are observed for the fixed p case. In Fig. 4(b), the cluster size distributions against s for $p = 0.15$ and several values of b are presented. For $b < b_c$, n_s shows a supercritical behavior; at $b = b_c$, n_s follows a power law; for $b > b_c$, n_s shows a subcritical behavior.

In general, a module means a set of nodes which are connected densely. In this definition of a module, a hierarchical network does not have modules in a general sense. However, if a module is redefined as a set of nodes which are affected by influential nodes, a module in a hierarchical network consists of a hub or directly connected hubs and the neighbors. Hence, in the hierarchical network with low p, the degree of a hub determines the size of a module. As the degree distribution follows a power-law, the size distribution of modules is also likely to follow a power-law. Since the payoff of the player on the node with small degree is likely to be smaller than that on the node with large degree, it is highly probable that the players in a module follow the strategy of a hub. Surely, the whole part of a module cannot become a cluster of permanent cooperators. In most cases, the part of a module forms the permanent cooperator cluster. Yet, as the size of a module becomes larger, the size of the permanent cooperator cluster is also likely to get larger. Therefore, the distribution of the size of permanent cooperator clusters also follows a power-law.

The networks constructed by the hierarchical network model are composed of mainly two types of motifs(Fig. 6). The motif consisting of only cooperator nodes

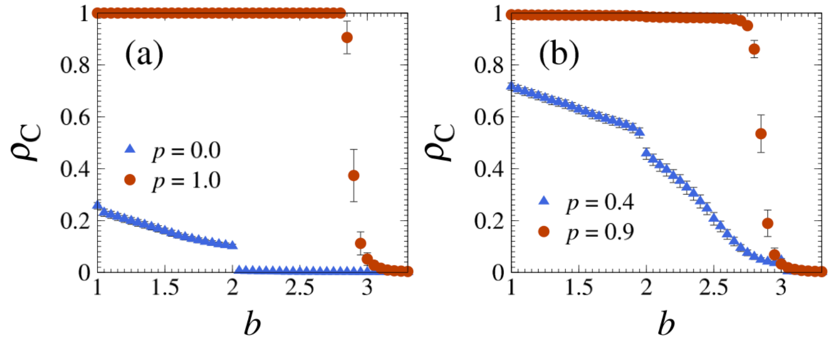


Figure 5: Density of cooperators as a function of temptation payoff b. (a) for the cases for $p = 0$ and $p = 1$, (b) for the cases for $p = 0.4$ and $p = 0.9$ in the hierarchical model

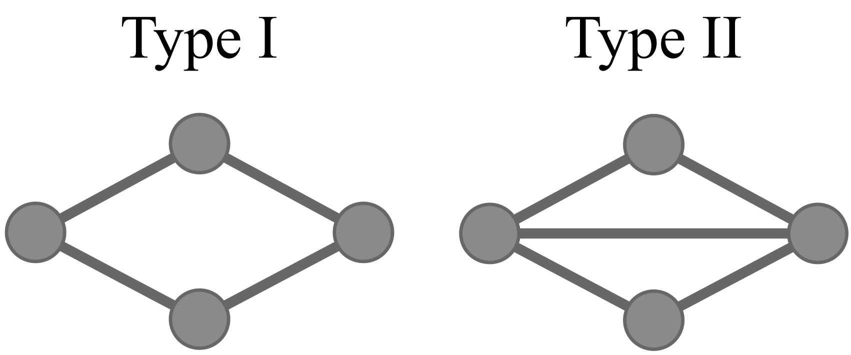


Figure 6: Hierarchical networks are composed of mainly two types of motifs.

can become the part of the cluster of permanent cooperators.

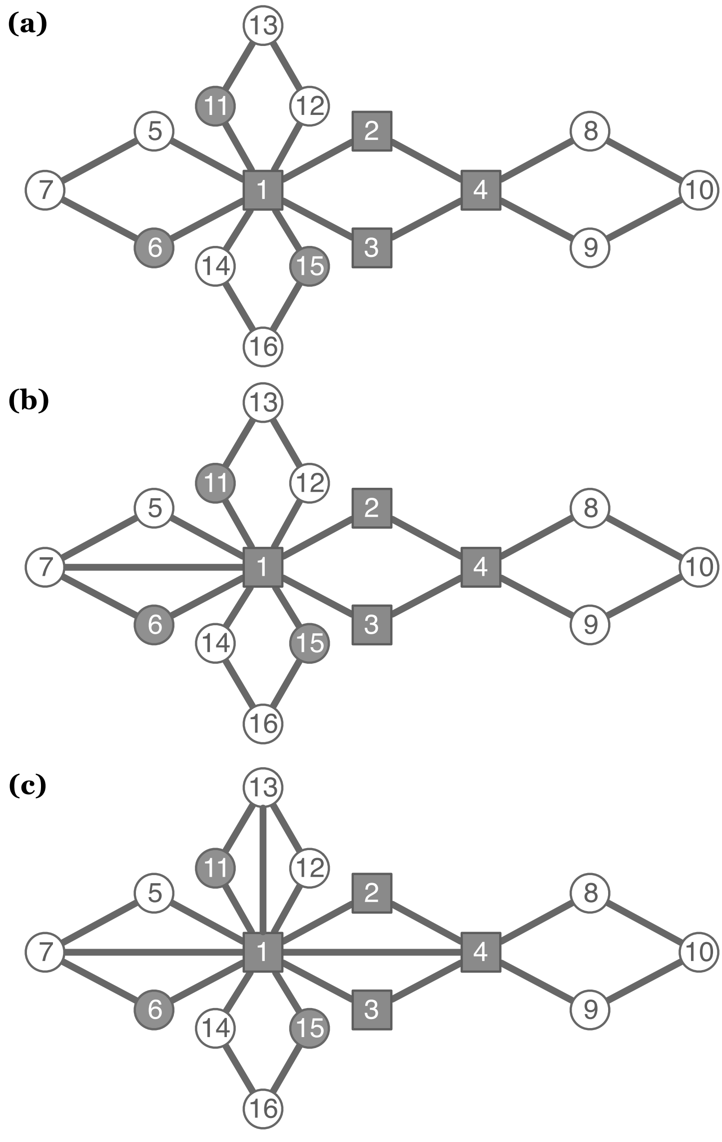


Figure 7: Cooperator clusters. Gray(white) nodes represent cooperators(defectors).

Cooperator clusters on several types of structures are depicted in Fig. 7. Gray nodes represent cooperators; white nodes are defectors. Player 1, 2, 3 and 4 which are cooperators reside in a motif of type I or II introduced in Fig. 6.

In Fig. 7(a), all motifs are type I. The nodes with degree more than 4 are connected only to the nodes with degree 2. These type of local structures are commonly observed in the hierarchical network of small p. Here, a node with degree more than 4 is called a local hub. Player 1 and 4 locate at local hubs. Player 2 and 3 locate at the node with degree 2. The payoffs of player 2 and 3 are 2, and those of player 1 and 4 equal or exceed 2. The payoff of player 8 is b. If $b \leq 2$, the cooperator cluster of player 1, 2, 3 and 4 is stable. For $b > 2$, the cluster is unstable, since the payoff of player 4 is lower than those of non-hub defectors. Therefore, for $b > 2$, the cooperator clusters of type-I motifs are vulnerable to defection. The density of the cooperator clusters of type-I motifs drops at $b = 2$. As a result, the density of cooperators drops at $b = 2$ for small p as shown in Fig. 5.

For $p > 0$, the strategies of local hubs can spread to other local hubs.

In Fig. 7(b) and (c), the strategy of player 1 can propagate to player 7 in the local hub. If player 5 and 7 become cooperators, the fixed income of player 1 will increase; the payoff of player 1 will be at least 5 in (b) and 6 in (c). The increase of the fixed income of a cooperator local hub increase raises the probability that the strategy of the cooperator local hub spread to the neighbors. Therefore, the connections between local hubs can induce the rise of the payoffs of cooperator local hubs and enhance the level of cooperation. The overall level of cooperation increases with p. However, this is not the case for $b > 3$. The cooperator cluster of type-II motif is unstable for $b > 3$. In Fig. 7(c), for $b > 3$, the payoff of player 4 is less than those of non-hub defectors in the neighborhood. If player 4 becomes a defector, the strategies of player 2 and 3 will fluctuate and the fixed income of player 1 will drop to zero. The density of the cooperator clusters of type-II drops at $b = 3$. Since the networks at $p = 1$ can be decomposed mainly into type-II motifs, the giant cluster size and the cooperator density ρ_C decrease suddenly at $b = 3$.

Recalling that the cooperator cluster of type-I motifs are unstable for $b > 2$, the fraction of cooperators drops close to 0 for $b > 3$ regardless of p as shown in Fig. 5.

The real networks are composed of various types of motifs, not only two types shown in Fig. 6. Therefore, the sudden drops of cooperation level at $b = 2$ and $b = 3$ are not expected to appear in the real networks.

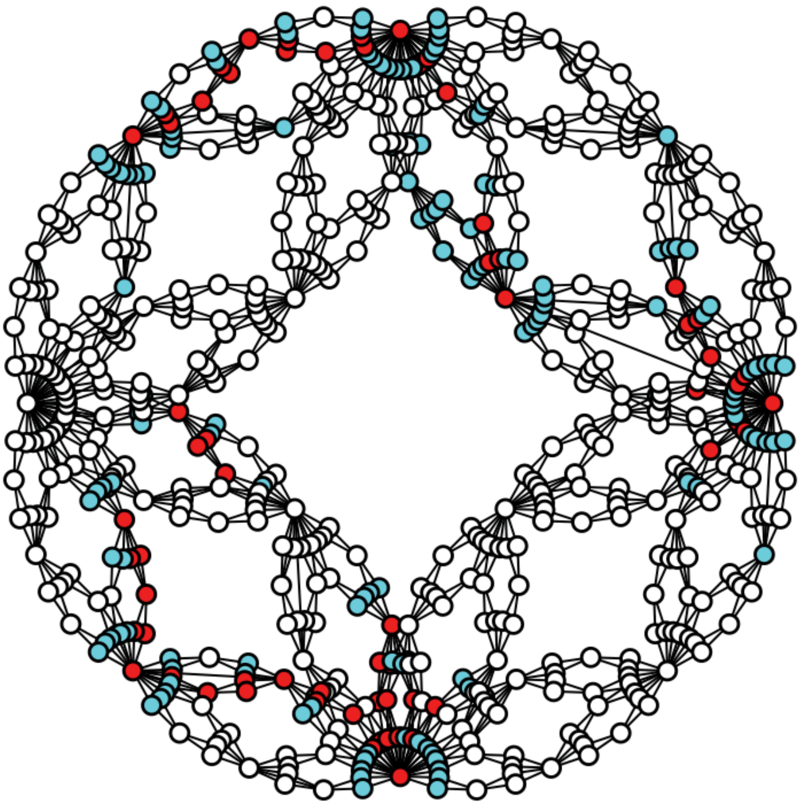


Figure 8: A snapshot of permanent cooperators(red), permanent defectors(white), and unstable players(cyan) in the hierarchical network with p=0.15, b=2.3, and N=684 after 20,000 rounds in a steady state

Shown in Fig. 8 is a snapshot depicting the state of the players when $p = 0.15$, $b = 2.3$ and $N = 684$. This network is close to a large-world network. The isolated cooperator cluster of a type-I motif cannot survive since $b > 2$. The cooperator clusters consist of solely type-II motifs, or the mixture of type-I and type-II motifs surrounded by type-I motifs. The cooperator can become a permanent cooperator when the payoff of a cooperator is guaranteed to be higher than those of neighbor defectors. To satisfy this condition, the cooperator needs the fixed income earned from the stable cooperator clusters. Hubs have advantages to satisfy this condition, since hubs have large degrees and the probability for hubs to belong to the stable cooperator clusters is relatively higher than that for other nodes. Surely non-hub nodes can also satisfy the conditions. The size of the cooperator cluster with hub cooperators is likely to be larger than that without hub cooperators. Here, a cooperator, even located in a hub, cannot survive if not belonging to the stable cooperator clusters. Cooperator clusters are formed with a wide range of sizes, because of the heterogeneity of the degree of the nodes and the stochastic process of the PD games

In the small-world network with large p, the strategies of the stable hub cooperators spread to the other hubs. The propagation of cooperation expands the sizes of cooperator clusters. The expansion of the clusters leads to the appearance of one giant cluster. Meanwhile, the propagation of strategies of hub cooperators to neighbors increase the fixed incomes of hubs as well. Therefore, permanent cooperators are most likely to locate at hubs.

2.4.2 Results on rewired hierarchical networks

The hierarchical network model is a good model for investigating a transition from large-world to small-world. The structure of hierarchical networks is, however, extremely regular. The networks are decomposed mainly into two types of motifs. The degree distributions are not continuous. Because of the structural regularity, the hierarchical network model may not be proper as a testbed to investigate the phenomena on the real networks. To overcome this weakness, we employ a link-rewiring process.

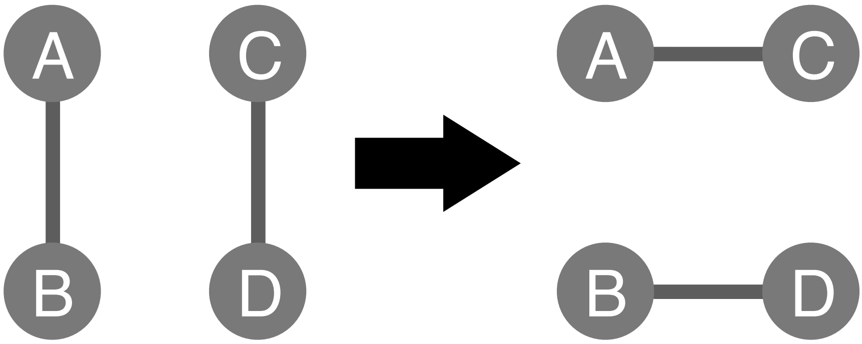


Figure 9: Description of link-rewiring process

Link-rewiring is one of the ways to transform a large-world network to a small-world network. A link-rewiring process consists of two steps; to select two links and to rewire two selected links, as shown in Fig. 9. In our study, two links are selected randomly. A link-rewiring process conserves the number of links and the degrees of all nodes. Rewiring of links of a network lessens the structural regularity as well.

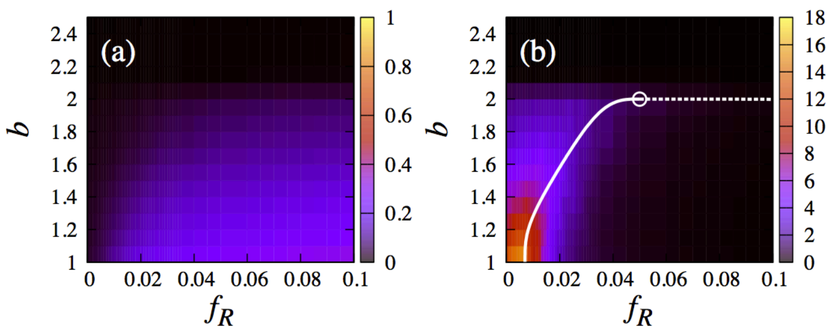


Figure 10: (a)Giant cluster size and (b)susceptibility for rewired networks. N=10,924. f_R is the fraction of rewired links.

We simulate the evolutionary prisoner's dilemma games on the rewired hierarchical networks. We construct the rewired network by rewiring f_R fraction of links of a base network. Here, we use a hierarchical network with $p = 0$ as a base network. The number of players is 10,924. Data are averages over 100 configurations.

First, we investigate the giant cluster size of the permanent cooperators(Fig. 10(a)). The giant cluster size reduces gradually with the increase of b regardless of f_R. The giant cluster size increases gradually with f_R regardless of b. This behavior differs from that in the hierarchical networks controlled by long-range bond probability p. Recalling that link-rewiring eases the structural regularity, the gradual transition of the giant cluster size is expected to be more acceptable and general than the abrupt transition of the giant cluster size in the hierarchical networks with p. Meanwhile, the networks for small f_R are composed of mainly type-I motifs in Fig. 6. The nodes in the network are of degree 2^m. By these two factors, for $b > 2$, cooperators die out, which may be not expected in the real networks.

The level of cooperation in the rewired networks appears to be low. Since the link-rewiring process does not guarantee the connection between hubs, the number of links connecting hubs is smaller in the rewired networks with f_R than in the hierarchical network with the comparable p. As presented in previous researches [38] [39], the connection between hubs plays a crucial role in the enhancement of cooperation.

Secondly, we investigate the susceptibility(Fig. 10(b)). There exists a tricritical-like point, which is the similar result with that of the games on the hierarchical model.

Next, we investigate the size distribution of permanent cooperator clusters for small f_R. As shown in Fig. 11, the distribution of cooperator cluster sizes exhibits a critical behavior at a certain value b_c for a given f_R.

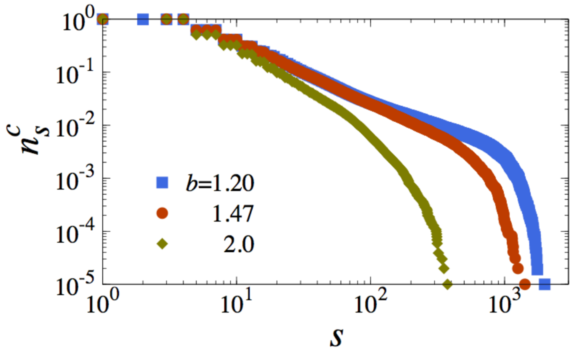


Figure 11: Accumulated size distribution of cooperator clusters for rewired networks at $f_R = 0.01$. f_R is the fraction of rewired links.

2.4.3 Results on the WWW network

So far, we investigated the characteristics of permanent cooperator clusters in the model networks. The results on the model networks are expected to reveal many traits of cooperation on the real large-world networks. Nevertheless, as pointed out in the previous sections, the extremely structural regularity of the model networks might cause some unrealistic results.

To compare the results on the model networks and the real-world ones, we simulate the evolutionary PD games on World-Wide Web(WWW). WWW is one of the well-known examples of the real large-world network [62] [63].

We use a part of the WWW composed of 325,729 [68]. This network is a scale-free network with degree exponent $\gamma \approx 2.6$. The fractal dimension is about 4.1, which is calculated by a box-covering method. It is known that the distribution of the module sizes in many real networks follows a power law [69] [70] [71] [72]. The cumulative distribution $P(S > s)$ of the module sizes of WWW also follow a power-law $P(S > s) \sim s^{-\nu}$ with the exponent $\nu \approx 1.15$ [73]. From the power-law behavior of the cumulative distribution, we can infer that the module size distribution follows a power-law $P(s) \sim s^{-\tau}$ with $\tau \sim 2.2$. Lancichinetti, *et. al* [74] investigated the overlapping and hierarchical module structure of networks, and reported that the size distribution of overlapping modules of WWW follows a power-law, and the value of the exponent is also close to 2.2.

WWW is a directed network, but the directionality of links is disregarded in our study.

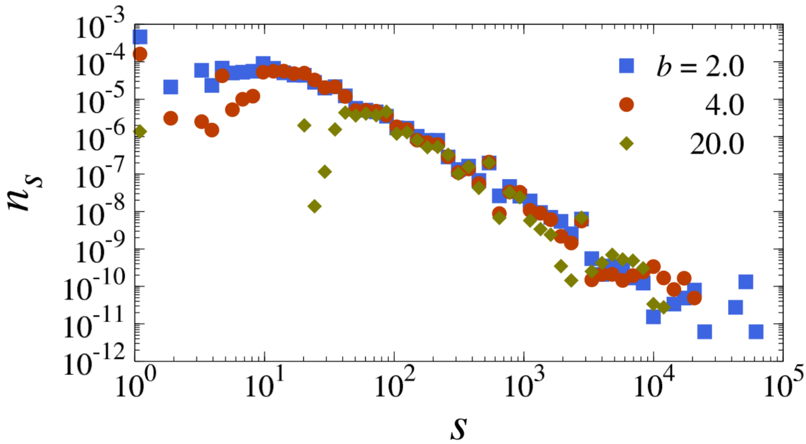


Figure 12: Size distribution of cooperator clusters in WWW

We investigate the distribution of cooperator cluster sizes for several b(Fig. 12). Data are averages over 100 configurations. The distribution of cluster sizes decays in a power-law manner with an exponent of approximately 2. This exponent is similar to the exponent τ of the module size distributions, which is approximately 2.2. This similarity originates from the module structure and the disassortativity of WWW. The payoffs of hubs are likely to be higher than those of non-hubs. The player with lower payoff cannot affect the player with higher payoff. Therefore, the probability that hubs affect non-hubs is higher than that of the opposite cases. The connections between hubs are few in WWW. The module in a fractal network is composed of a few connected hubs and the non-hubs around the hubs. A hub affluences the players within the neighborhood directly and within the module indirectly. The initial strategies of hubs are likely to determine the strategies of players in a module. Consequently the sizes of cooperator cluster are compatible with the sizes of the modules. On the other hand, there are some differences among the size distributions for bs. The numbers of small clusters decrease with increasing b. The increase of b raises the probability that the hub cooperators have smaller payoffs than the non-hub defectors. The degrees of hubs are low in small modules relative to in large modules. Hence the resistance to defectors is likely to be less in small modules than in large ones. The sizes of large clusters decrease with the increase of b. For small b, large modules and small modules in the neighborhood can be connected and act like a mega-module. For large b, the strategies of the players connecting modules change continuously, and the modules are separated by these players.

The critical behaviors of the distribution of the cluster sizes are observed in the PD games on the hierarchical networks for small p, the rewired hierarchical networks for small f_R and WWW. These model networks and the large-world networks in real world have the common characteristics of the modular structure and the disassortativity. These types of behaviors are expected in the games on other real large-world networks.

In the result on the hierarchical network, there exists a specific value of b above which all the cooperator clusters vanish. This is caused by the local structure of the hierarchical network. The hierarchical network consists of mainly two types of motifs which are vulnerable to the invasion of defection above the specific value of b. The abrupt destruction of cooperator clusters, however, is not observed in WWW. In WWW, the number of cooperator clusters decreases continuously with b. This means that the resistance to defection varies between modules. It is expected to be extremely rare that a real network is composed of only a few type of motifs. Therefore, in most of real fractal scale-free networks, the number of cooperator clusters may decrease with the increase of b.

2.5 Summary

After the advent of the concept of scale-free networks, PD games on these heterogeneous networks have been studied, and some researchers claimed that these types of networks promote the level of cooperation. Most of the researches studied PD games of on small-world scale-free networks. In small-world scale-free networks, as the hubs usually have higher payoffs or fitnesses than the neighbors do, the neighbors imitate the strategies of hubs. When a hub has the strategy of cooperation, the neighbors change their strategies to cooperation, and the high payoff of the hub can be maintained steadily. When the strategy of a hub is defection, the neighbors is likely to have the strategy of defection, which lowers the payoff of the hub. Therefore, the defective hubs are likely to have lower payoff than the cooperative hubs do. One of the characteristics of small-world scale-free networks is the connection among hubs. The strategy of cooperator hubs can propagate to defector hubs, and this grows the sizes of cooperator clusters, which lead to the appearance of a giant cooperator cluster.

The real networks such as WWW and protein-interaction networks, however, are found to be fractal networks, of which the characteristics are the rare connection among hubs and the modular structure. As the connection among hubs is one of the main factors that raise the level of cooperation, the PD games on fractal scale-free networks differ from those on small-world scale-free networks. In this study, we focused on the difference of the formation of cooperator clusters on both type of networks.

We followed the game rules suggested by Santos, *et al*, in which they claimed that the level of cooperation rises on scale-free networks. As the spatial structure, a hierarchical model was used. In this model, by varying the fraction of long-range links, we can control the property of a network from large-world to small-world.

In the large-world networks, a variety of cooperator clusters with various sizes are formed. This is because the structure of a fractal scale free network is modular, and the sizes of modules vary. The rare connection among hubs hinders the propagation of strategies between clusters. Therefore the advent of a giant cooperator cluster is not expected in fractal scale-free networks. The fraction of cooperators is lower in a fractal network than in a small-world network.

The increase of the value of temptation or the fraction of long-range links leads to the decrease of the variety of cooperator cluster sizes. If the value of temptation rises, the survivability of small cooperator clusters is lowered.

The variety of cooperator cluster sizes and the low level of cooperation are also observed in the PD games on WWW which is one of the real fractal networks, and are expected to appear in the games on other real fractal networks.

Chapter 3

Evolutionary stability in the spatial evolutionary PD games with mixed-strategies

3.1 Introduction to mixed strategies

So far, we studied the PD games with only two strategies, cooperation and defection. If a player chooses the cooperation strategy, the player cooperates with all of neighbors until the strategy of the player changes. The player with strategy of defection defects all of neighbors. The existence of unconditional cooperators or defectors is, however, somewhat unrealistic.

The introduction of cooperation probability can generalize the games. A strategy of a player is determined by the cooperation probability P^C from 0 to 1. For example, the cooperation probability of a player is 0.6, then the player cooperates with the probability of 60%, and defects with 40%. The cooperation probability of 1 is mapped to the cooperation strategy in the standard PD games. The cooperation probability of 0 is equivalent to the defection strategy. Here, the cooperation probability is the only factor which determines the player's action.

In game theory, the strategy which mixes more than two pure strategies with certain probabilities is called a *mixed strategy*. In PD games, cooperation and defection are pure strategies.

The mixed strategy is an essential element in the game theory. Consider each player has a certain strategy. If each player has no benefit from changing its strategy, while the other players keep their strategies unchanged, then the set of strategies is Nash equilibrium. Nash [75] [76] proved that every finite games have at least one Nash equilibrium. Some games however do not have Nash equilibria where all players have pure strategies. These games have mixed-strategy Nash equilibria. A typical example is Rock-Scissors-Paper game, where Nash equilibrium is that each player has the mixed strategy of 1/3 rock, 1/3 scissors, and 1/3 paper.

BitTorrent, which is a protocol for file sharing is one of the real-world examples which the mixed strategy prisoner's dilemma game can be applied to. The BitTorrent protocol allows users to upload and download the files simultaneously. A file is divided into *pieces* with the same sizes. By transferring the pieces, the simultaneous upload and download of a file are enabled. A node which downloads and uploads the pieces of a file is called a *peer*. A *seeder* is a node which uploads the file after completing the download of a file. A peer downloads the pieces from seeds and peers. The number of the pieces of a single file is predetermined, and this is the number of connection which a peer need. One of the aims of this protocol is the decentralized distributions of files.

A peer(seeder) can control the speed limit of upload and download. Lowering the speed limit of upload can enhance the speed of download. Hence, a rational peer lowers the upload speed. The download speed will, however, become slower eventually if all the peers and seeders are rational and they lower the speed of upload. This situation is similar to the prisoner's dilemma. The limit speed of upload can be interpreted as a cooperation level. In this sense, the transfer of a file in BitTorrent protocol can be the mixed-strategy prisoner's dilemma.

Aside the example of BitTorrent, there are many behaviors and phenomena which can be explained by the game with mixed strategies. In the field of physics, however, the games with mixed strategies have been rarely studied.

3.1.1 Payoffs in mixed-strategy PD games

Once cooperation probability P^C of players are determined, we can calculate the expectation value of payoffs. In this study, the payoff matrix with T=b, R=1, P=S=0 is used. For example, consider the mixed-strategy PD game($b > 1$) with an A-type player with cooperation probability P_A^C and a B-type player with P_B^C. The expected payoff of A-type player, $\text{Payoff}_{A \leftarrow B}$ and that of B-type player, $\text{Payoff}_{B \leftarrow A}$ are represented by

$$\text{Payoff}_{A \leftarrow B} = P_B^C [P_A^C + b(1 - P_A^C)] \tag{3.1}$$

$$\text{Payoff}_{B \leftarrow A} = P_A^C [P_B^C + b(1 - P_B^C)]. \tag{3.2}$$

In this study, the expectation value of payoff from games is used as a payoff. The difference between two players' payoffs is calculated as follows.

$$\begin{aligned} &\text{Payoff}_{B \leftarrow A} - \text{Payoff}_{A \leftarrow B} \\ &= P_A^C [P_B^C + b(1 - P_B^C)] - P_B^C [P_A^C + b(1 - P_A^C)] \\ &= b(P_A^C - P_B^C) \end{aligned} \tag{3.3}$$

When $P_B^C < P_A^C$, the payoff of B-type player is higher than that of A-type player. Hence, in the mixed-strategy PD games with two players, the payoff of the player with lower P^C is higher.

3.2 Evolutionary stability in PD game with mixed strategies

The introduction of mixed strategies into spatial games generalizes the games, and can make the games more realistic. Nevertheless, the spatial games with mixed strategies have been very rarely studied. In this study, we introduce the spatial PD games with mixed strategies. As a first step, we study the evolutionary stability in the spatial evolutionary PD games with mixed strategies.

The evolutionarily stable strategy is defined as follows. Consider all players in a group have strategy A. When a player with strategy B come into this group, can this player survive or strategy B invade into the group? If the answer is yes, strategy A is not evolutionarily stable to the strategy B. In the other case, strategy A is evolutionarily stable to the strategy B.

In general, an evolutionary stable strategy(ESS) is a strategy which, if all players have this strategy, does not allow the invasion of any other (initially rare) strategy. Once all the players in a group have an ESS, the group is resistant to the occasional invasion of other strategy. Maynard Smith and Price firstly introduced the concept of evolutionary stability [77]. Evolutionary stability does not explain why and how a certain strategy in the real world becomes a major strategy. Instead, this concept tells whether a major strategy can be stable under the attacks of other strategies.

Maynard Smith and Price suggested two conditions for an ESS. Consider the game with two strategies, T, S. For strategy S to be an evolutionary stable strategy, strategy S should satisfy one of the following conditions.

1. $\text{Payoff}_{S\leftarrow S} > \text{Payoff}_{T\leftarrow S}$
2. $\text{Payoff}_{S\leftarrow S} = \text{Payoff}_{T\leftarrow S}$ and $\text{Payoff}_{S\leftarrow T} > \text{Payoff}_{T\leftarrow T}$

In a group where most of players have a strategy S, if the payoff of player with strategy T from the game with players with strategy S is smaller than other player's payoff, strategy T vanishes by natural selection. This is the meaning of the first condition.

If $\mathrm{Payoff}_{S \leftarrow S} = \mathrm{Payoff}_{T \leftarrow S}$, the number of player with strategy T can increase. Nevertheless, if $\mathrm{Payoff}_{S \leftarrow T} > \mathrm{Payoff}_{T \leftarrow T}$, the average payoff of player with strategy T is likely to be smaller than that of player with strategy S, which is a disadvantage of strategy T in a competition with strategy S. This is the meaning of the second condition.

If certain animal and human behavioral patterns and social norms have been sustained for a long time, then these might be the products of evolutionarily stable strategies. Therefore, many researchers studied the evolutionary stability [1] [78] in a variety of disciplines. Most studies on evolutionary stability, however, did not consider the spatial structure. There are only a few studies on the evolutionary stability in the spatial evolutionary games [79]. These studies, however, did not consider the mixed strategies.

The invasion of initially rare strategy in PD game on networks can be an analogue of the propagation of an idea, a meme and an opinion in structured population. Considering this analogy, some research subjects in physics, such as opinion model, voter model and epidemic model are similar to evolutionary stability. Since we study the influence of one mutant on the equilibrium state, our study can be seen as the damage spreading for spatial PD games with mixed strategies.

In the PD games with mixed strategies, the number of strategies can be more than three; theoretically infinite. In this study the number of strategies in a single game is limited to two. Initially all the players except one player have strategy A, and only one player has strategy B. We simulate the PD games with this initial configuration and measure the fraction of players with strategy B in a stable state. To confirm the evolutionary stability of strategy A, we investigate the fraction of players with strategy B', varying B'. If strategy A does not allow the invasion of any other strategies, strategy A is evolutionary stable.

The evolutionary stable strategies in the original sense do not allow the invasion of other strategies at all; no player with initially rare strategies survives. In this study, we weaken the condition for ESS; ESS in our sense is the strategy which do not allow the invasion of other strategies above a specified fraction f_B^-. If there is at least one strategy with which the fraction surpasses f_B^-, strategy A is not an ESS. In this study, f_B^- is set to 1/100.

We use several types of regular graphs as the spatial structure for PD games. It is found that under a deterministic rule, there always exists the evolutionary stable strategies.

3.3 Rules of games

The initial strategies of all the players but one are strategy A. Players with strategy A cooperate with probability P_A^C. Only one randomly chosen player has strategy B. This player has cooperation probability P_B^C.

In this study, an asynchronous update rule is used. At one simulation time step, only one randomly chosen player has the chance to change its strategy.

Table 3: Two rules for reference selection

	Reference selection rule
Rule I	random reference selection
Rule II	deterministic reference selection

We use two types of reference selection rules, as shown in Table 3. Rule I is a random reference selection rule. A candidate player chooses one of neighbors randomly. Rule II is a deterministic reference selection rule. A candidate selects a neighbor with the highest fitness as a reference neighbor. We simulate PD games under a random rule and a deterministic rule separately.

Adoption probability is defined as follows. If the fitness of a candidate is lower than that of a reference neighbor, the candidate adopts the strategy of the reference. If the fitness of the candidate is higher, the candidate keeps its own strategy. If the fitnesses of two players equal, the candidate adopts the strategy of the reference with probability of 0.5.

Payoff matrix is a simplified version with T=*b*, R=1, P=S=0. The expected value of a payoff from a game with a neighbor is used as a payoff from the game.

$$\mathrm{Payoff}_{A\leftarrow A} = P_A^C[P_A^C + b(1-P_A^C)] \tag{3.4}$$

$$\mathrm{Payoff}_{A\leftarrow B} = P_B^C[P_A^C + b(1-P_A^C)] \tag{3.5}$$

$$\mathrm{Payoff}_{B\leftarrow A} = P_A^C[P_B^C + b(1-P_B^C)] \tag{3.6}$$

$$\mathrm{Payoff}_{B\leftarrow B} = P_B^C[P_B^C + b(1-P_B^C)] \tag{3.7}$$

As a fitness of a player, we use the average payoff, which is the sum of payoffs divided by the number of neighbors.

We simulate the evolutionary PD games by iterating the following steps; the selection of a candidate player, the selection of a reference neighbor, the adoption of strategy and the re-calculation of fitnesses.

3.4 Fitnesses of players

In this study, an average payoff is used as the fitness of a player. Let us compare the fitnesses of an A-type player and a B-type players. A player with cooperation probability P_A^C is an A-type player and one with (P_B^C) is a B-type player.

An A-type player has the neighbors of A-type players and B-type players. The number of A-type players among the neighbors of a A-type player is k_A^A. Likewise, the number of B-type player among the neighbors of a A-type player is k_A^B. Similarly, k_B^A and k_B^B are defined. Then, the fraction of players among the neighbors of a player can be defined as

$$\begin{aligned} f_A^A &= \frac{k_A^A}{k_A}, f_A^B = \frac{k_A^B}{k_A}, \\ f_B^A &= \frac{k_B^A}{k_B}, f_B^B = \frac{k_B^B}{k_B}, \end{aligned} \tag{3.8}$$

where k_A is the number of neighbors of an A-type player and k_B is that of an A-type player.

The fitness F_A of a A-type player and the fitness F_B of a B-type player are calculated by

$$\begin{aligned} F_A &= f_A^A \mathrm{Payoff}_{A \leftarrow A} + f_A^B \mathrm{Payoff}_{A \leftarrow B} \\ &= f_A^A P_A^C (P_A^C + b(1 - P_A^C)) + f_A^B P_B^C (P_A^C + b(1 - P_A^C)) \end{aligned} \tag{3.9}$$

$$\begin{aligned} F_B &= f_B^A \mathrm{Payoff}_{B \leftarrow A} + f_B^B \mathrm{Payoff}_{B \leftarrow B} \\ &= f_B^A P_A^C (P_B^C + b(1 - P_B^C)) + f_B^B P_B^C (P_B^C + b(1 - P_B^C)) \\ &= f_B^A P_A^C [P_B^C + b(1 - P_B^C)] + (1 - f_B^A) P_B^C [P_B^C + b(1 - P_B^C)]. \end{aligned} \tag{3.10}$$

In the case of $f_B^A \neq 1$, the condition that the fitness of a B-type player is higher than that of a A-type player is

$$min(P_A^C, X) < P_B^C < max(P_A^C, X), \tag{3.11}$$

where

$$X = -\frac{f_A^A}{1-f_B^A} P_A^C + \frac{b(f_A^A - f_B^A)}{(b-1)(1-f_B^A)}, \tag{3.12}$$

and $min(P_A^C, X)$ is the smallest value between P_A^C and$X$, and $max(P_A^C, X)$ is the largest value.

In the case of $f_B^A = 1$, the condition that the fitness of a B-type player is higher than that of a A-type player is $P_B^C < P_A^C$.

3.4.1 Fitnesses in regular graphs

A regular graph is the graph where all the players have the same number of neighbors. Consider a games for $P_B^C < P_A^C$ on a regular graph with degree n. In the PD game on regular graphs, the following relations are always satisfied [80].

$$F_X^s > F_X^{s-1} \ (s \in \{1, ..., n\}) \tag{3.13}$$

$$F_B^n = max\{F_A^s, F_B^s\} \tag{3.14}$$

$$F_A^0 = min\{F_A^s, F_B^s\} \tag{3.15}$$

$$F_B^s > F_A^s \ (s \in \{0, ..., n\}) \tag{3.16}$$

Here, F_X^s is the fitness of X player with which k_X^A is s, and X is A or B.

3.5 Evolutionary stability on complete graphs

A complete graph is a graph in which every pair of nodes is connected. In the complete graph with N nodes, each node has $N-1$ neighbors and the number of links is $N(N-1)/2$.

Let us consider the PD game with $N-1$ players with strategy A and one player with strategy B. The initial fitness of a A-type player is F_A^{N-2}, and that of a B-type player is F_B^{N-1}. For $P_B^C < P_A^C$, the fitness of a B-type player is higher than those of A-type players by Eq. 3.13 and Eq. 3.16, so that a B-type player survives and strategy B spreads to other nodes. For $P_B^C > P_A^C$, B-type player does not survive.

Consider that the number of the remaining A-type player is n for $P_B^C < P_A^C$. The fitness of a A-type player is F_A^{n-1} and that of a B-type player is F_B^n. Since the fitness of a B-type player is always higher than that of an A-type player, strategy B propagates to all the players at last.

In short, if a new strategy is more cooperative than the existing one of the other players, then the invasion of the new strategy is not possible. The strategy which is less cooperative than the existing one, however, propagates to all the players. Hence, the only evolutionary stable strategy is the strategy with $P^C = 0$.

3.6 Evolutionary stability on regular graph with degree 2

As a first step to study the evolutionary stability on regular graph, we study the evolutionary PD games on the regular graph with degree 2. We use cycle graphs.

A cycle graph is a graph composed of a single cycle, as shown in Fig. 13. A cycle graph with N nodes has N links. The degree of a node is 2.

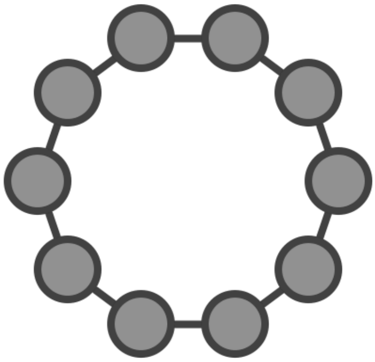


Figure 13: A cycle graph with N=10

3.6.1 Comparison between fitnesses of two players with different strategies

There are two types of players, A and B. An A-type player has cooperation probability P_A^C, and a B-type player has P_B^C. The conditions for $F_A < F_B$ differ for $P_A^C > P_B^C$ and $P_A^C < P_B^C$. The conditions are presented in Table 4 for $P_A^C > P_B^C$, and in Table 5 for $P_A^C < P_B^C$.

Table 4: Conditions for $F_A < F_B$ in the regular graph with degree 2 for $P_A^C > P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	$0 < P_B^C < P_A^C$
1/2	0	$-\frac{1}{2}P_A^C + \frac{1}{2}\frac{b}{b-1} < P_B^C < P_A^C$
1	0	$-P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	1/2	$-\frac{b}{b-1} < P_B^C < P_A^C$
1/2	1/2	$-P_A^C < P_B^C < P_A^C$
1	1/2	$-2P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0, 1/2, 1	1	$P_B^C < P_A^C$

With these conditions, the parameter space of (P_A^C, P_B^C) can be divided into sections. The sections for b=1.5, 2.0 and 3.0 are shown in Fig. 14, Fig. 15 and Fig. 16, respectively. In a section, the inequality of $\{F_A^s,\ F_B^s\}(s = 0,1,2)$ is same

Table 5: Conditions for $F_A < F_B$ in the regular graph with degree 2 for $P_A^C < P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	impossible
1/2	0	$P_A^C < P_B^C < -\frac{1}{2}P_A^C + \frac{1}{2}\frac{b}{b-1}$
1	0	$P_A^C < P_B^C < -P_A^C + \frac{b}{b-1}$
0	1/2	impossible
1/2	1/2	impossible
1	1/2	$P_A^C < P_B^C < -2P_A^C + \frac{b}{b-1}$
0, 1/2, 1	1	impossible

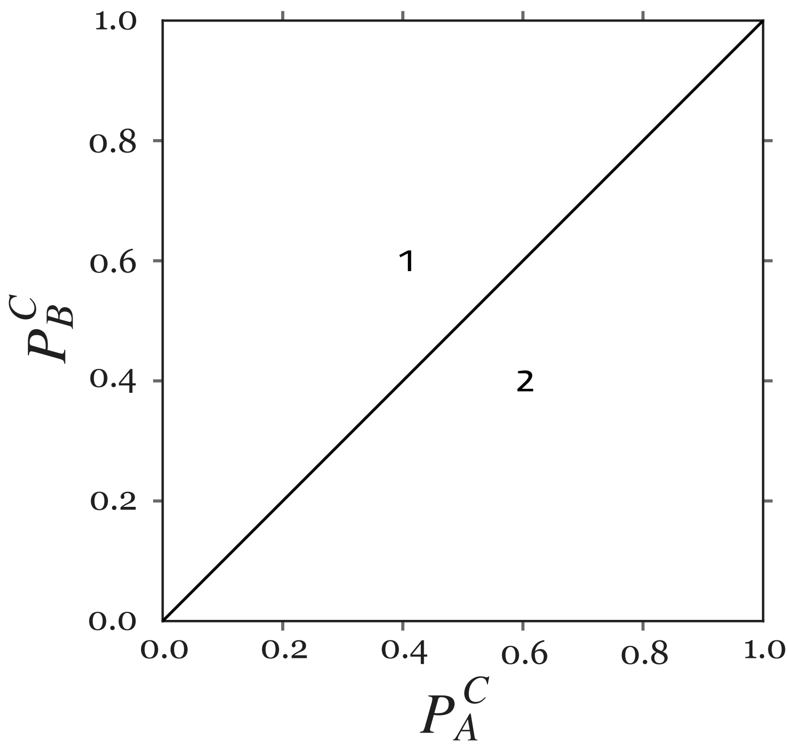


Figure 14: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=1.5 for the regular graphs with degree 2. Here $s = 0, 1, 2$

Table 6: The inequalities of fitnesses in sections of Fig. 14.

section	inequalities of fitnesses
1	$F_B^2 < F_A^2 < F_B^1 < F_B^0 < F_A^1 < F_A^0$
2	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_B^2$

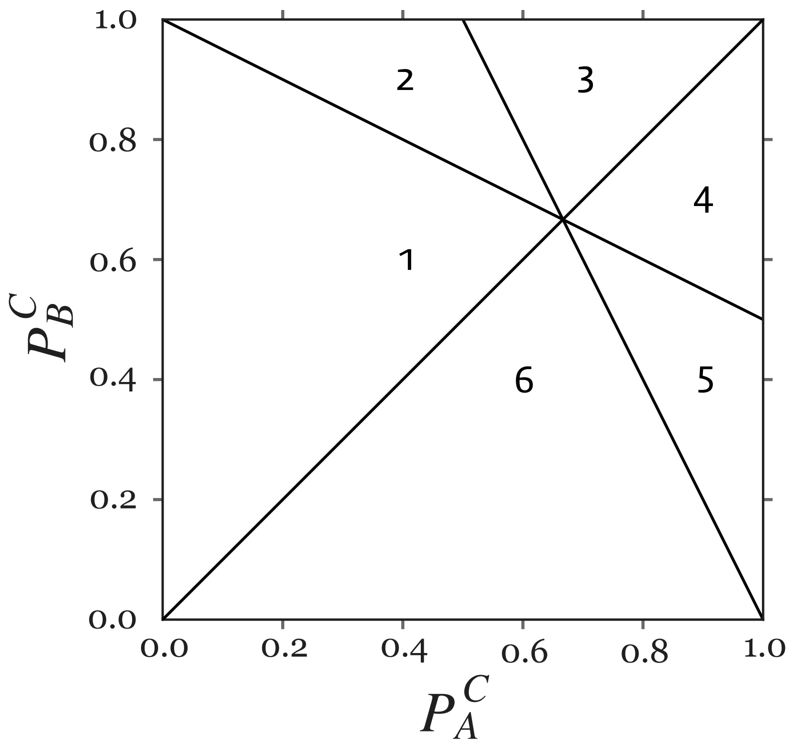


Figure 15: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=2.0 for the regular graphs with degree 2. Here $s = 0, 1, 2$.

Table 7: The inequalities of fitnesses in sections of Fig. 15.

section	inequalities of fitnesses
1	$F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^2 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
4	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_B^1 < F_B^2$
5	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_B^1 < F_B^2$
6	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2$

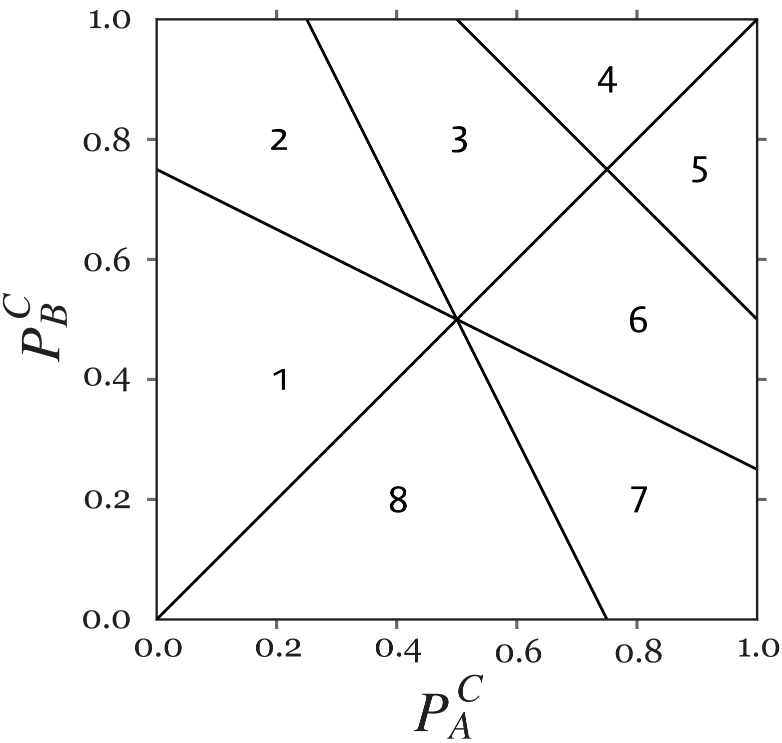


Figure 16: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=3.0 for the regular graphs with degree 2. Here $s = 0, 1, 2$.

Table 8: The inequalities of fitnesses in sections of Fig. 16

section	inequalities of fitnesses
1	$F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^2 < F_A^2 < F_B^1 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
4	$F_B^2 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
5	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_B^1 < F_B^2$
6	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_B^1 < F_B^2$
7	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_B^2$
8	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2$

regardless of P_A^C and P_B^C. For example, the inequality of fitnesses at section 2 in Fig. 14 is $F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2$.

Table 9: Section indices with the same inequality of fitnesses for b=3.0, 2.0, 1.5.

b=3.0	b=2.0	b=1.5
1	1	1
2	2	-
3	3	-
4	-	-
5	-	-
6	4	-
7	5	-
8	6	2

3.6.2 Simulation results and discussions

We simulate the mixed-strategy PD games on cycle graph of $N = 1000$ for several bs under two reference selection rules, and measure the fraction of player$B$. The results are shown in Fig. 17. Data are averages over 200 configurations. Black lines separate the sections based on the inequalities of fitnesses. The results under Rule I are shown in shown in Fig. 17 (a), (c), (e), and those under Rule II in (b), (d), (f).

The results under Rule I are same regardless of the value of b. For $P_A^C < P_B^C$, the

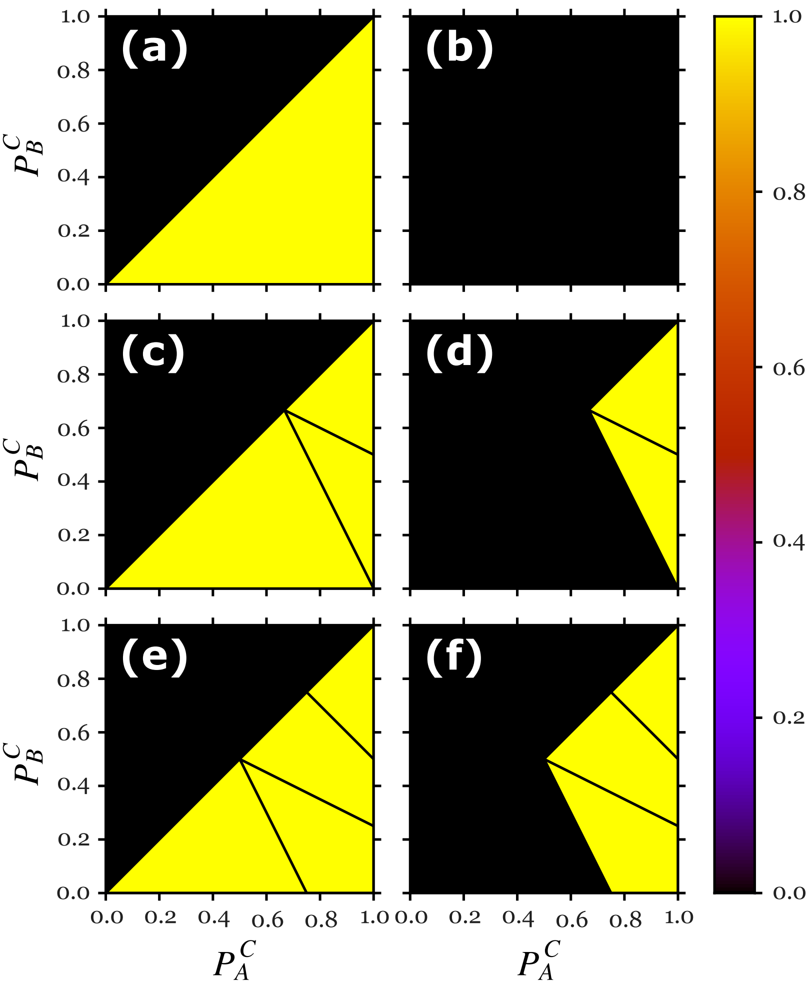


Figure 17: The fraction of B-type players in the mixed-strategy PD games on the cycle graph of $N = 1000$. (a), (c), (e) Rule I, (b), (d), (f) Rule II. (a), (b) b=1.5, (c), (d) b=2.0, (e), (f) b=3.0

A-type players do not allow the invasion of B-type players regardless of the value of b. As $F_A^1 > F_B^2$, a B-type player adopts the strategy of neighbors and strategy B vanishes.

For $P_A^C > P_B^C$, strategy B propagates to all the players, and strategy A vanishes. A strategy allows the invasion of other strategies with smaller P^C. Hence, no the strategies except the one with $P^C = 0$ are evolutionary stable under Rule I.

For $P_A^C < P_B^C$ under Rule II, strategy B cannot invade. No B-type player survives.

The results for $P_A^C > P_B^C$ under Rule II differ from those under Rule I. For $b = 1.5$, strategy A blocks the invasion of strategy B. As the value of b increases, the area where strategy B invades widens in the parameter space. In this area, all the players adopt strategy B.

The areas where strategy B cannot invade under Rule II for b=1.5, 2.0 and 3.0 correspond to section 2 of Fig. 14, section 6 of Fig. 15, section 8 of Fig. 16, respectively. These sections have the same order of fitnesses(Table 9). Under Rule II, only two B-type players survive in these sections. The fraction of B-type players is 2/N, which goes to 0 in the limit $N \to \infty$.

For $b = 1.5$, all strategies are evolutionary stable. For $b = 2.0$, the strategy with $P^C \leq 2/3$ are evolutionary stable. For $b = 3.0$, the strategy with $P^C \leq 1/2$ are the evolutionarily stable strategies. From the calculation, we obtain that for arbitrary b, the strategy with $P^C \leq \frac{b}{3(b-1)}$ are evolutionary stable. Here, by definition, P^C has the value from 0 to 1. Therefore, for $b \leq 3/2$, all the strategies are evolutionary stable.

The behavior of propagation in each section is analyzed in Appendix A.

3.7 Evolutionary stability on regular graph with degree 3

In this section, the evolutionary stability on regular graph with degree 3 is studied.

3.7.1 Comparison between fitnesses of two players with different strategies

The conditions for $F_A < F_B$ are presented in Table 10 for $P_A^C > P_B^C$ and in Table 11 for $P_A^C < P_B^C$.

Table 10: Conditions for $F_A < F_B$ in the regular graph with the degree of 3 for $P_A^C > P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	$0 < P_B^C < P_A^C$
1/3	0	$-\frac{1}{3}P_A^C + \frac{b}{3(b-1)} < P_B^C < P_A^C$
2/3	0	$-\frac{2}{3}P_A^C + \frac{2b}{3(b-1)} < P_B^C < P_A^C$
1	0	$-P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	1/3	$-\frac{b}{2(b-1)} < P_B^C < P_A^C$
1/3	1/3	$-\frac{1}{2}P_A^C < P_B^C < P_A^C$
2/3	1/3	$-P_A^C + \frac{b}{2(b-1)} < P_B^C < P_A^C$
1	1/3	$-\frac{3}{2}P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	2/3	$-\frac{2b}{b-1} < P_B^C < P_A^C$
1/3	2/3	$-P_A^C - \frac{b}{b-1} < P_B^C < P_A^C$
2/3	2/3	$-2P_A^C < P_B^C < P_A^C$
1	2/3	$-3P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0, 1/3, 2/3, 1	1	$P_B^C < P_A^C$

3.7.2 Simulation results and discussions

We investigate the faction of B-type players in the mixed-strategy PD games on two types of regular graphs with degree 3 for several bs under two reference selection rules. The each results on the honeycomb lattice and random regular graph are shown in Fig. 21 and Fig. 22, respectively. Data are averages over 200 configura-

Table 11: Conditions for $F_A < F_B$ in the regular graph with the degree of 3 for $P_A^C < P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	impossible
1/3	0	$P_A^C < P_B^C < -\frac{1}{3}P_A^C + \frac{b}{3(b-1)}$
2/3	0	$P_A^C < P_B^C < -\frac{2}{3}P_A^C + \frac{2b}{3(b-1)}$
1	0	$P_A^C < P_B^C < -P_A^C + \frac{b}{(b-1)}$
0	1/3	impossible
1/3	1/3	impossible
2/3	1/3	$P_A^C < P_B^C < -P_A^C + \frac{b}{2(b-1)}$
1	1/3	$P_A^C < P_B^C < -P_A^C + \frac{b}{2(b-1)}$
0	2/3	impossible
1/3	2/3	impossible
2/3	2/3	impossible
1	2/3	$P_A^C < P_B^C < -3P_A^C + \frac{b}{(b-1)}$
0, 1/3, 2/3, 1	1	impossible

Table 12: The inequalities of fitnesses in sections at Fig. 18

section	inequalities of fitnesses
1	$F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
3	$F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
4	$F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
5	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_B^1 < F_B^2 < F_B^3$
6	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_B^1 < F_B^2 < F_B^3$
7	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
8	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_B^3$

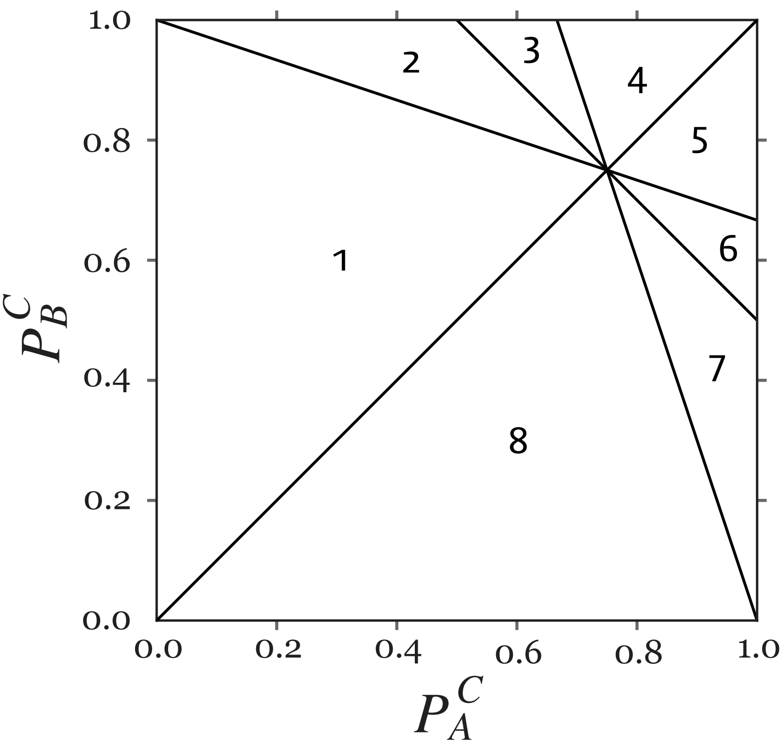


Figure 18: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=1.5 for the regular graphs with degree 3. Here, $s = 0, 1, 2, 3$.

Table 13: The inequalities of fitnesses in sections at Fig. 19

section	inequalities of fitnesses
1	$F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^3 < F_A^3 < F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
4	$F_B^3 < F_B^2 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
5	$F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
6	$F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
7	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_B^1 < F_B^2 < F_B^3$
8	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_B^1 < F_B^2 < F_B^3$
9	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
10	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
11	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_B^2 < F_B^3$
12	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_B^3$

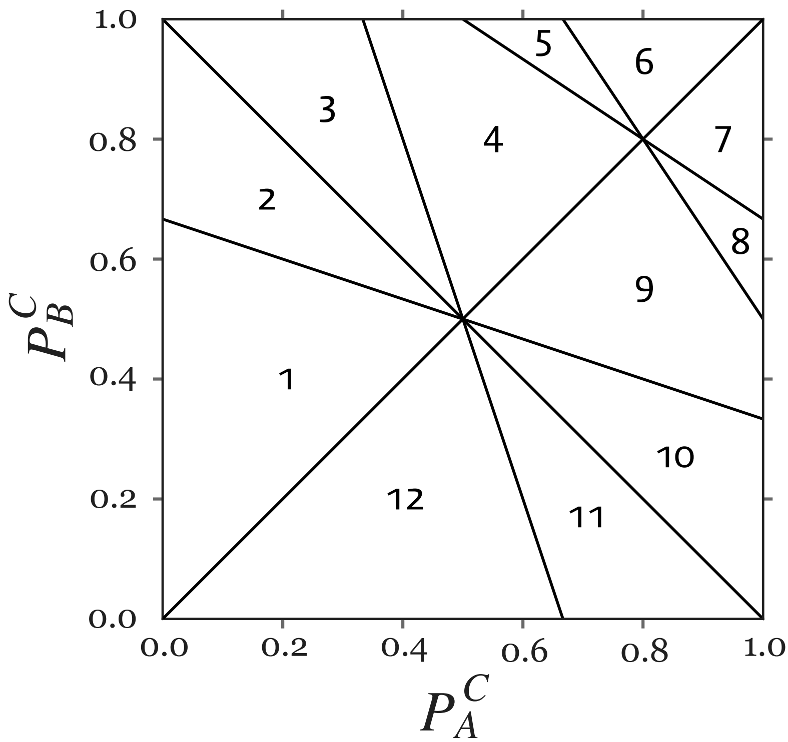


Figure 19: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=2 for the regular graphs with degree 3. Here, $s = 0, 1, 2, 3$.

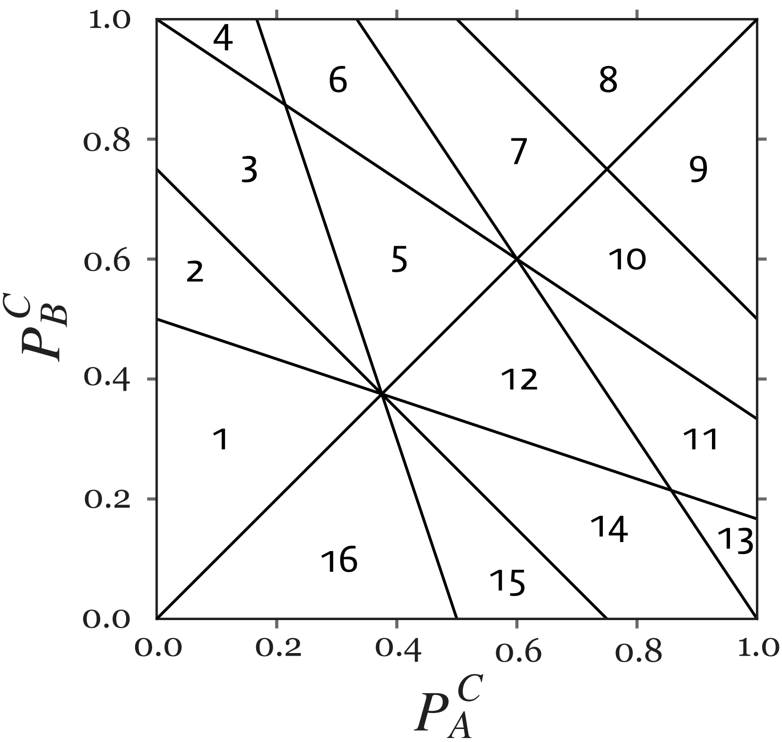


Figure 20: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=3 for the regular graphs with degree 3. Here, $s = 0, 1, 2, 3$.

Table 14: The inequalities of fitnesses in sections at Fig. 20

section	inequalities of fitnesses
1	$F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^3 < F_A^3 < F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
4	$F_B^3 < F_A^3 < F_B^2 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
5	$F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
6	$F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
7	$F_B^3 < F_B^2 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
8	$F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
9	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_B^1 < F_B^2 < F_B^3$
10	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
11	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
12	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_B^1 < F_A^3 < F_B^2 < F_B^3$
13	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_A^3 < F_B^1 < F_B^2 < F_B^3$
14	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_B^2 < F_B^3$
15	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_A^3 < F_B^2 < F_B^3$
16	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_B^3$

Table 15: Section indices with the same inequality of fitnesses for b=3.0, 2.0, 1.5.

b=3.0	b=2.0	b=1.5
1	1	1
2	2	2
3	3	3
4	-	-
5	4	4
6	5	-
7	6	-
8	-	-
9	-	-
10	7	-
11	8	-
12	9	5
13	-	-
14	10	6
15	11	7
16	12	8

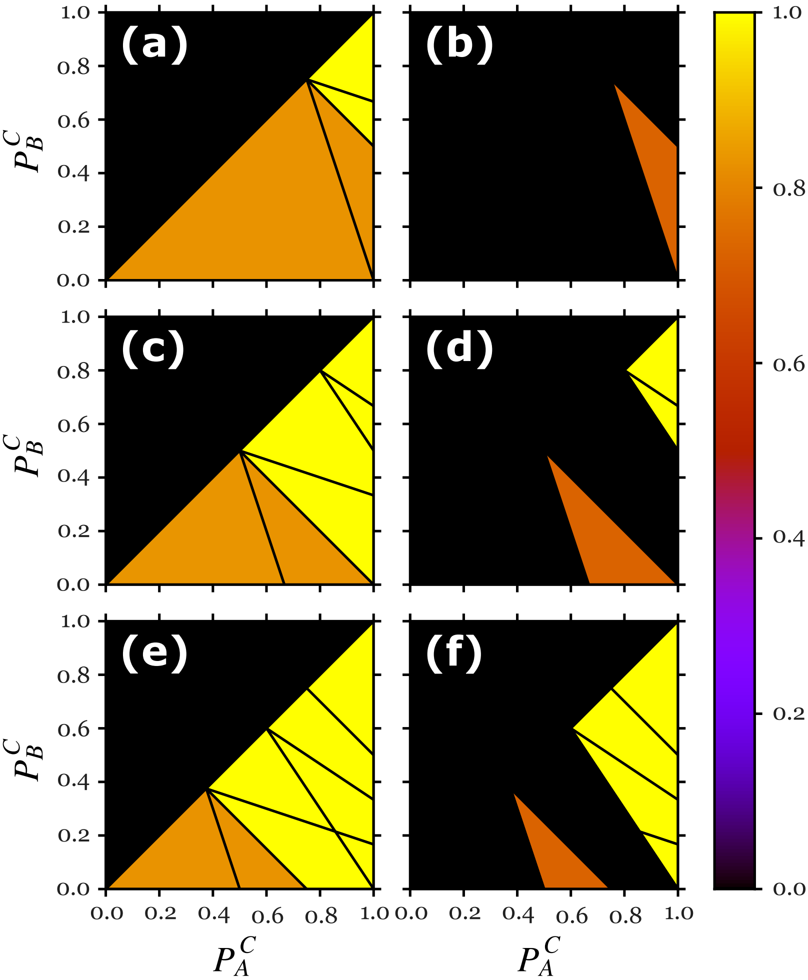


Figure 21: The fraction of B-type players in the mixed-strategy PD games on the honeycomb lattice of $N = 9800$. (a), (c), (e) Rule I, (b), (d), (f) Rule II. (a), (b) b=1.5, (c), (d) b=2.0, (e), (f) b=3.0

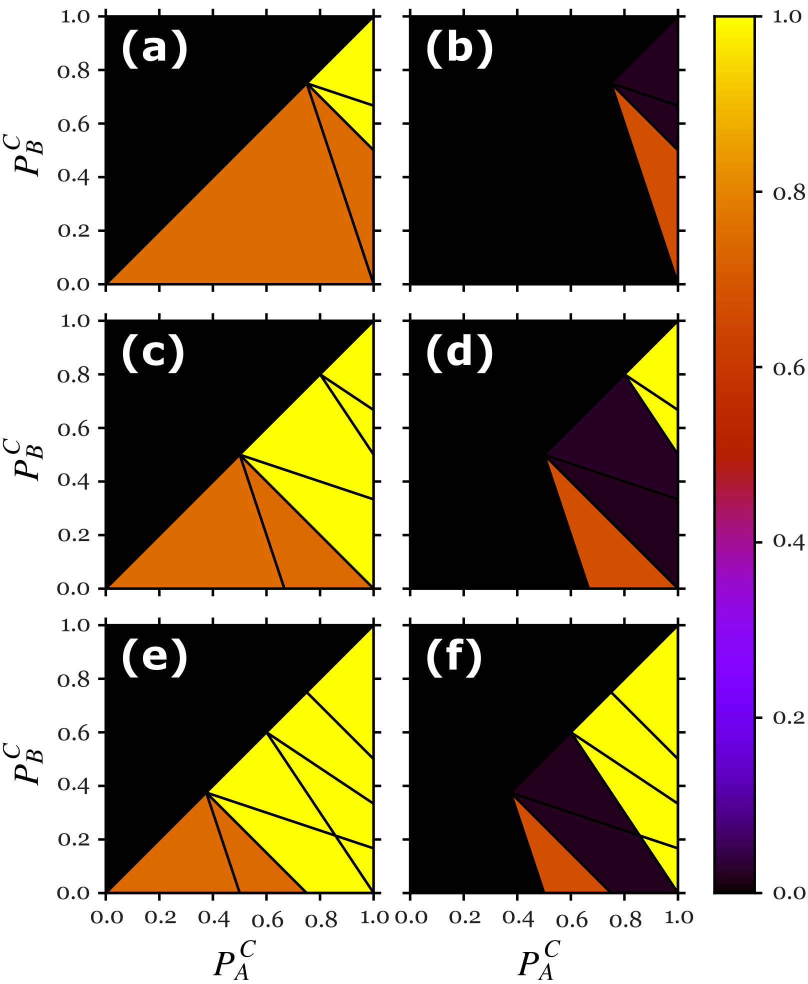


Figure 22: The fraction of B-type players in the mixed-strategy PD games on the random regular graphs of $N = 10000$ with degree 3. (a), (c), (e) Rule I, (b), (d), (f) Rule II. (a), (b) b=1.5, (c), (d) b=2.0, (e), (f) b=3.0

tions. The number of players on honeycomb lattice is 9800, and that on the random regular graph is 10000.

The propagation behaviors on two types of graphs are similar in parameter space(P_A^C, P_B^C) for the same b and the same reference selection rule. The only differences are the fractions of B-type players in the sections where strategy A and B coexist.

For $P_A^C < P_B^C$, strategy A does not allow the invasion of strategy B for any bs on any graphs regardless of the reference selection rule. As $F_A^2 > F_B^3$, a B-type player adopts the strategy of neighbors and strategy B vanishes.

For $P_A^C > P_B^C$, however, the propagation behaviors are different depending on the reference selection rule.

Under Rule I, strategy B invades with a considerable fraction. Strategy A vanishes or coexists with strategy B. Therefore, under Rule I, the strategy with $P^C = 0$ is the only evolutionarily stable strategy.

Under Rule II, there exists the sections where the invasion of strategy B is not allowed. Strategy with $P^C \leq P^{C*}$ can block the invasion of other strategy. Here, P^{C*} is 0.75 for $b = 1.5$, 0.5 for $b = 2$, and 0.375 for $b = 3$. From calculation, we obtain $P^{C*} = \frac{b}{4(b-1)}$. Hence, for $b \leq 4/3$, all the strategy is evolutionary stable.

In mixed-strategy PD games with two players, lowering P_B^C, while fixing P_A^C, raises the payoff of a B-type player. Similarly, the payoff of a B-type player increase with P_A^C for fixed P_B^C. Higher payoffs are likely to lead to higher fitnesses, since a fitness is defined as the average of payoffs. The relatively higher fitnesses of B-type players, however, do not always lead to the the higher fraction of B-type players. For example, for $b = 3$, the fitnesses of B-type players in section 12 and 14 are relatively higher than those in section 15. Nonetheless, in the game with Rule II, the fraction of B-type players in section 12 and 14 is lower than that in section 15.

More detailed results on honeycomb lattice are in Appendix B and those on random regular graphs with degree 3 are Appendix D.

3.8 Evolutionary stability on regular graph with degree 4

In this section, we study the evolutionary stability on the graphs with degree 4.

3.8.1 Comparison between fitnesses of two players with different strategies

The conditions for $F_A < F_B$ are presented in Table 16 for $P_A^C > P_B^C$ and in Table 17 for $P_A^C < P_B^C$.

3.8.2 Simulation results and discussions

We investigate the faction of B-type players in the mixed-strategy PD games on two types of regular graphs with degree 4 for several bs under two reference selection rules. The results on the square lattice and random regular graph are shown in Fig. 21 and Fig. 22, respectively. Data are averages over 200 configurations. The number of players is 10000 on both graphs.

On the whole, the propagation behaviors on two types of graphs are similar in parameter space(P_A^C, P_B^C) for the same b and the same reference selection rule. However, unlike the results on the regular graphs with degree 3, there exist the sections where the propagation behaviors are remarkably different depending on the spatial structures. For example, in section 21and 24 for $b = 3.0$, all players adopt strategy B in the games on square lattice. However, in the same sections, the final fraction of B-type players is not 1in the games on random regular graph of

Table 16: Conditions for $F_A < F_B$ in the regular graph with the degree of 4 for $P_A^C > P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	$0 < P_B^C < P_A^C$
1/4	0	$-\frac{1}{4}P_A^C + \frac{b}{4(b-1)} < P_B^C < P_A^C$
2/4	0	$-\frac{1}{2}P_A^C + \frac{b}{2(b-1)} < P_B^C < P_A^C$
3/4	0	$-\frac{3}{4}P_A^C + \frac{3b}{4(b-1)} < P_B^C < P_A^C$
1	0	$-P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	1/4	$-\frac{b}{3(b-1)} < P_B^C < P_A^C$
1/4	1/4	$-\frac{1}{3}P_A^C < P_B^C < P_A^C$
2/4	1/4	$-\frac{2}{3}P_A^C + \frac{b}{3(b-1)} < P_B^C < P_A^C$
3/4	1/4	$-P_A^C + \frac{2b}{3(b-1)} < P_B^C < P_A^C$
1	1/4	$-\frac{4}{3}P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	2/4	$-\frac{b}{b-1} < P_B^C < P_A^C$
1/4	2/4	$-\frac{1}{2}P_A^C - \frac{b}{2(b-1)} < P_B^C < P_A^C$
2/4	2/4	$-P_A^C < P_B^C < P_A^C$
3/4	2/4	$-\frac{3}{2}P_A^C + \frac{b}{2(b-1)} < P_B^C < P_A^C$
1	2/4	$-2P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0	3/4	$-\frac{3b}{(b-1)} < P_B^C < P_A^C$
1/4	3/4	$-P_A^C - \frac{2b}{b-1} < P_B^C < P_A^C$
2/4	3/4	$-2P_A^C - \frac{b}{b-1} < P_B^C < P_A^C$
3/4	3/4	$-3P_A^C < P_B^C < P_A^C$
1	3/4	$-4P_A^C + \frac{b}{b-1} < P_B^C < P_A^C$
0, 1/4, 2/4, 3/4, 1	1	$P_B^C < P_A^C$

Table 17: Conditions for $F_A < F_B$ in the regular graph with the degree of 4 for $P_A^C < P_B^C$.

f_A^A	f_B^A	Conditions for $F_A < F_B$
0	0	impossible
1/4	0	$P_A^C < P_B^C < -\frac{1}{4}P_A^C + \frac{b}{4(b-1)}$
2/4	0	$P_A^C < P_B^C < -\frac{1}{2}P_A^C + \frac{b}{2(b-1)}$
3/4	0	$P_A^C < P_B^C < -\frac{3}{4}P_A^C + \frac{3b}{4(b-1)}$
1	0	$P_A^C < P_B^C < -P_A^C + \frac{b}{b-1}$
0	1/4	impossible
1/4	1/4	impossible
2/4	1/4	$P_A^C < P_B^C < -\frac{2}{3}P_A^C + \frac{b}{3(b-1)}$
3/4	1/4	$P_A^C < P_B^C < -P_A^C + \frac{2b}{3(b-1)}$
1	1/4	$P_A^C < P_B^C < -\frac{4}{3}P_A^C + \frac{b}{b-1}$
0	2/4	impossible
1/4	2/4	impossible
2/4	2/4	impossible
3/4	2/4	$P_A^C < P_B^C < -\frac{3}{2}P_A^C + \frac{b}{2(b-1)}$
1	2/4	$P_A^C < P_B^C < -2P_A^C + \frac{b}{b-1}$
0	3/4	impossible
1/4	3/4	impossible
2/4	3/4	impossible
3/4	3/4	impossible
1	3/4	$P_A^C < P_B^C < -4P_A^C + \frac{b}{b-1}$
0, 1/4, 2/4, 3/4, 1	1	impossible

Table 18: The inequalities of fitnesses in sections at Fig. 23

section	sorted list of fitnesses
1	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_B^0 < F_A^1 < F_A^0$
2	$F_B^4 < F_A^4 < F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
3	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
4	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_A^4 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
5	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^4 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
6	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_A^4 < F_B^0 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
7	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
8	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
9	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
10	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_A^4 < F_B^3 < F_B^4$

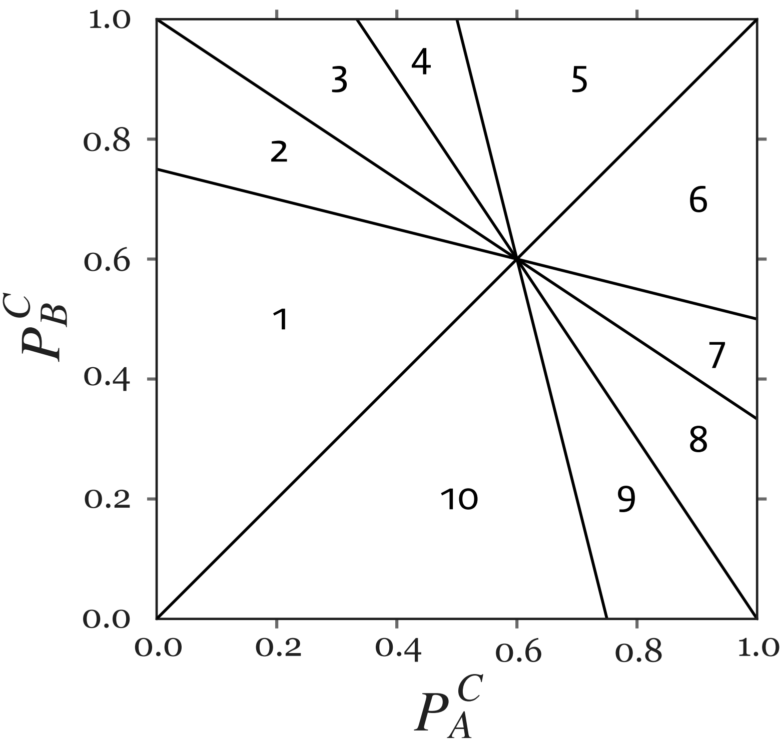


Figure 23: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=1.5 for the regular graphs with degree of 4. Here, $s = 0, 1, 2, 3, 4$.

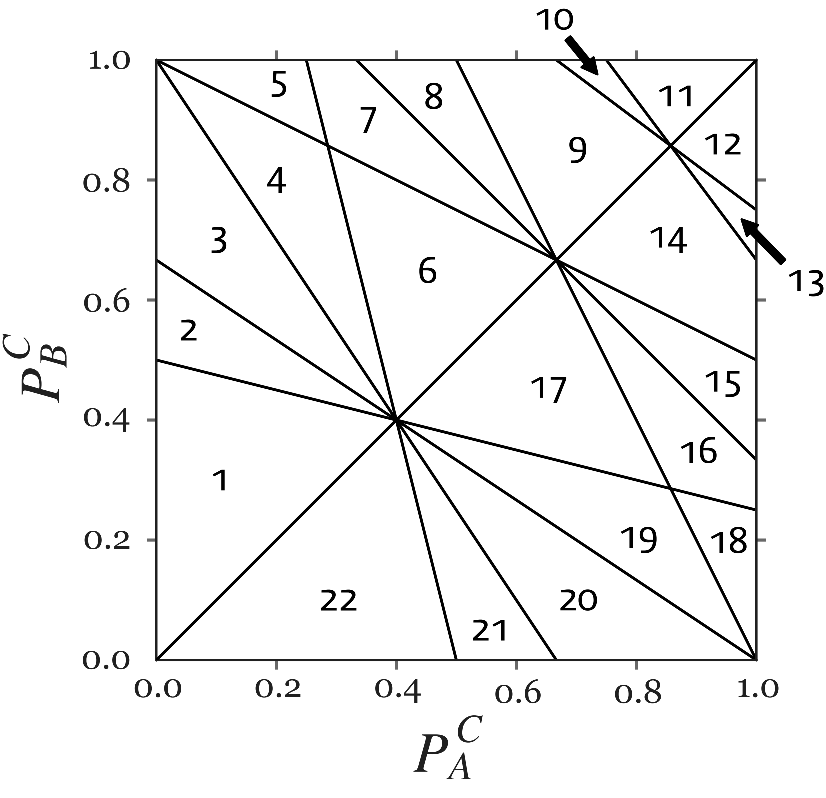


Figure 24: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=2 for the regular graphs with degree of 4. Here, $s = 0, 1, 2, 3, 4$.

Table 19: The inequalities of fitnesses in sections at Fig. 24

section	inequalities of fitnesses
1	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^4 < F_A^4 < F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
4	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
5	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
6	$F_B^4 < F_B^3 < F_B^2 < F_A^4 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
7	$F_B^4 < F_B^3 < F_B^2 < F_A^4 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
8	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_A^4 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
9	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^4 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
10	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^4 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
11	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^4 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
12	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_A^4 < F_B^0 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
13	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_A^4 < F_B^0 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
14	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_A^4 < F_B^0 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
15	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
16	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
17	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_B^1 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
18	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
19	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_B^1 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
20	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
21	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_A^3 < F_B^2 < F_A^4 < F_B^3 < F_B^4$
22	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_B^3 < F_A^4 < F_B^4$

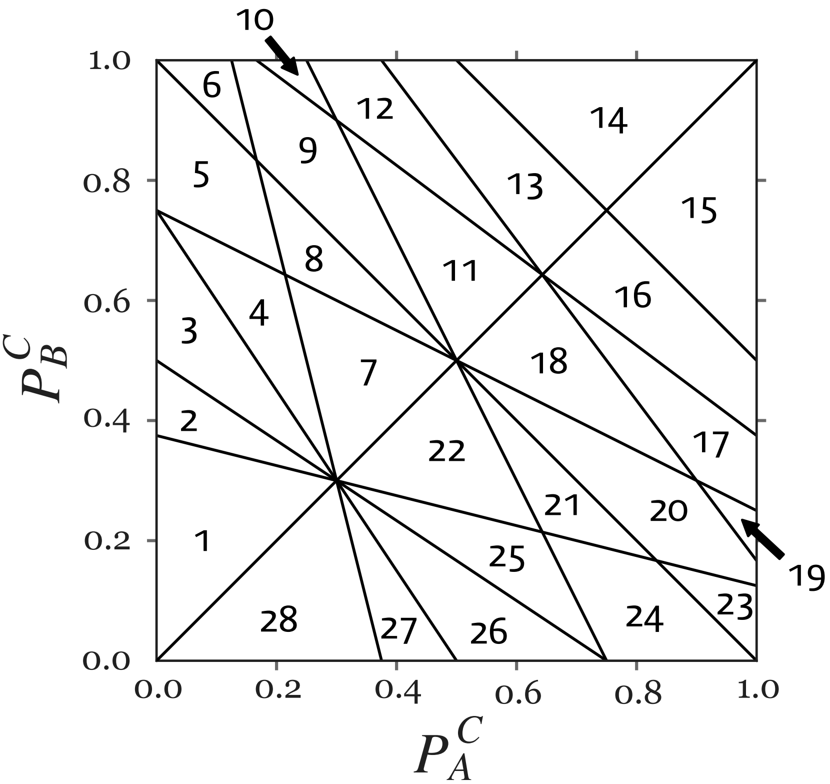


Figure 25: Sections divided on the inequalities of $\{F_A^s, F_B^s\}$ at b=3 for the regular graphs with degree of 4. Here, $s = 0, 1, 2, 3, 4$.

Table 20: The inequalities of fitnesses in sections at Fig. 20

section	inequalities of fitnesses
1	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_A^1 < F_B^0 < F_A^0$
2	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_A^2 < F_B^1 < F_B^0 < F_A^1 < F_A^0$
3	$F_B^4 < F_A^4 < F_B^3 < F_A^3 < F_B^2 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
4	$F_B^4 < F_A^4 < F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
5	$F_B^4 < F_A^4 < F_B^3 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
6	$F_B^4 < F_A^4 < F_B^3 < F_B^2 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
7	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_A^3 < F_B^1 < F_A^2 < F_B^0 < F_A^1 < F_A^0$
8	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_A^3 < F_B^1 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
9	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
10	$F_B^4 < F_B^3 < F_A^4 < F_B^2 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
11	$F_B^4 < F_B^3 < F_B^2 < F_A^4 < F_B^1 < F_A^3 < F_B^0 < F_A^2 < F_A^1 < F_A^0$
12	$F_B^4 < F_B^3 < F_B^2 < F_A^4 < F_B^1 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
13	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_A^4 < F_B^0 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
14	$F_B^4 < F_B^3 < F_B^2 < F_B^1 < F_B^0 < F_A^4 < F_A^3 < F_A^2 < F_A^1 < F_A^0$
15	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_A^4 < F_B^0 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
16	$F_A^0 < F_A^1 < F_A^2 < F_A^3 < F_B^0 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
17	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
18	$F_A^0 < F_A^1 < F_A^2 < F_B^0 < F_A^3 < F_B^1 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
19	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_A^4 < F_B^1 < F_B^2 < F_B^3 < F_B^4$
20	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_A^3 < F_B^1 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
21	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_B^1 < F_A^3 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
22	$F_A^0 < F_A^1 < F_B^0 < F_A^2 < F_B^1 < F_A^3 < F_B^2 < F_A^4 < F_B^3 < F_B^4$
23	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_A^3 < F_B^1 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
24	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_A^4 < F_B^2 < F_B^3 < F_B^4$
25	$F_A^0 < F_B^0 < F_A^1 < F_A^2 < F_B^1 < F_A^3 < F_B^2 < F_A^4 < F_B^3 < F_B^4$
26	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_A^3 < F_B^2 < F_A^4 < F_B^3 < F_B^4$
27	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_A^4 < F_B^3 < F_B^4$
28	$F_A^0 < F_B^0 < F_A^1 < F_B^1 < F_A^2 < F_B^2 < F_A^3 < F_B^3 < F_A^4 < F_B^4$

Table 21: Section indices with the same inequality of fitnesses for *b*=3.0, 2.0, 1.5.

b=3.0	b=2.0	b=1.5
1	1	1
2	2	2
3	3	3
4	4	4
5	5	-
6	-	-
7	6	5
8	7	-
9	8	-
10	-	-
11	9	-
12	10	-
13	11	-
14	-	-
15	-	-
16	12	-
17	13	-
18	14	-
19	-	-
20	15	-
21	16	-
22	17	6
23	-	-
24	18	-
25	19	7
26	20	8
27	21	9
28	22	10

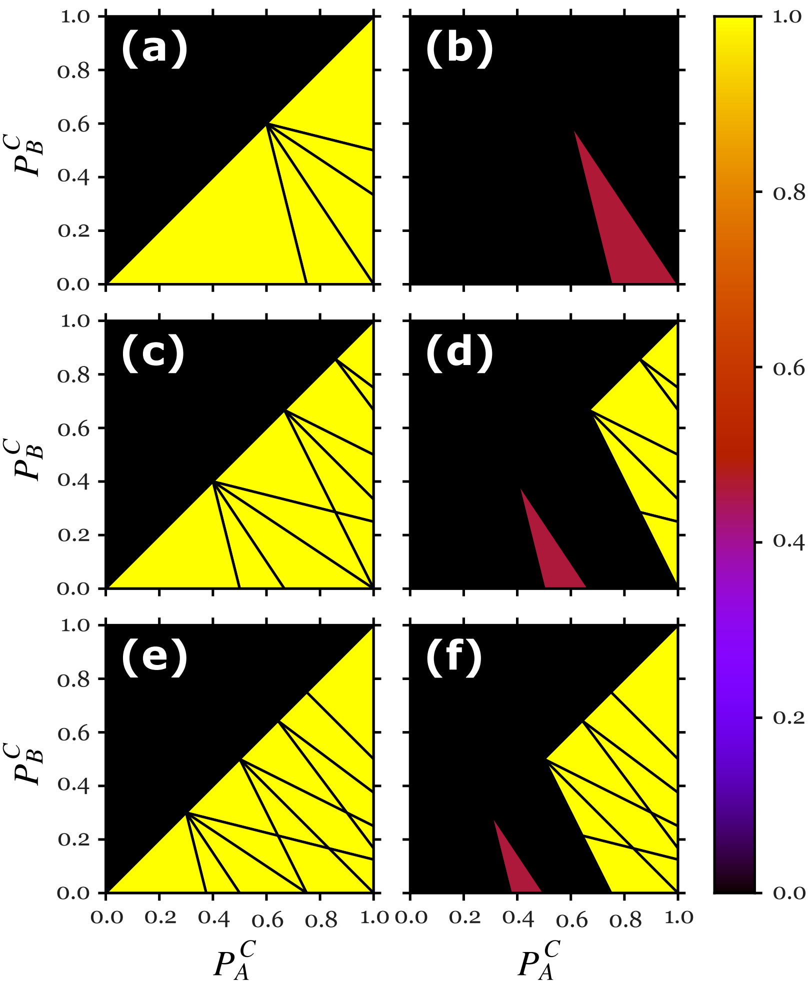


Figure 26: The fraction of B-type players in the mixed-strategy PD games on the square lattice of $N = 1000$. (a), (c), (e) Rule I, (b), (d), (f) Rule II. (a), (b) b=1.5, (c), (d) b=2.0, (e), (f) b=3.0

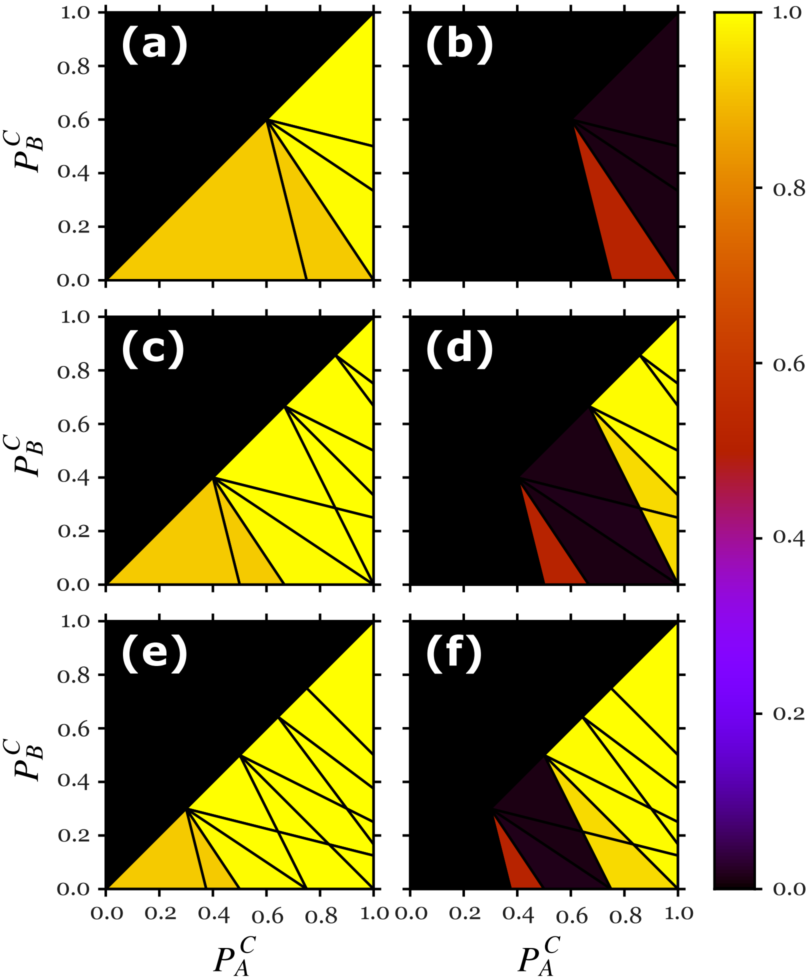


Figure 27: The fraction of B-type players in the mixed-strategy PD games on the random regular graphs of $N = 10000$ with degree 4. (a), (c), (e) Rule I, (b), (d), (f) Rule II. (a), (b) b=1.5, (c), (d) b=2.0, (e), (f) b=3.0

degree 4, and small fraction of A-type players can survive.

For $P_A^C < P_B^C$, strategy A does not allow the invasion of strategy B for any bs on any graphs regardless of the reference selection rule. As $F_A^3 > F_B^4$, a B-type player adopts the strategy of neighbors and strategy B vanishes.

For $P_A^C > P_B^C$, however, the propagation behaviors are different depending on the reference selection rule.

Under Rule I, strategy B always invades with a considerable fraction. Strategy A vanishes or coexists with strategy B. Hence, under Rule I, no strategies except the one with $P^C = 0$ are evolutionary stable.

Under Rule II, there exist the sections where strategy B cannot propagate. Strategy with $P^C \leq P^{C^*}$ blocks the invasion of other strategy. Here, P^{C^*} is 0.6 for $b = 1.5$, 0.4 for $b = 2$, and 0.3 for $b = 3$. From calculation, we obtain $P^{C^*} = \frac{b}{5(b-1)}$. Hence, for $b \leq 5/4$, all the strategy is evolutionary stable.

Similar to the cases in the games on regular graphs with degree 3, the relatively higher fitnesses of B-type players do not always increase the fraction of B-type players. For example, the fraction of B-type players are lower in section 22, 25, and 26 than in section 27.

More detailed results on square lattice are in Appendix C and those on random regular graphs with degree 4 are Appendix E.

3.9 Discussions

In the games with Rule I, a randomly selected neighbor's fitness is the only information which a candidate player needs. Under Rule II, a candidate needs more amount of informations. The candidate should know the fitnesses of all neighbors. Therefore, we can say that a player under Rule II is more prudent than a player under Rule I.

Under Rule I, all the strategies are vulnerable to other strategies with lower P^C. The group of players with the high level of cooperation can collapse into that with the very low level of cooperation. It is even possible that the group with $P^C = 1$ can be turned into that with $P_C = 0$.

Under Rule II, the behaviors differ. For $b \leq b^*$, all the strategies are evolutionarily stable. The cooperation level of a group can be maintained, since the strategy with lower cooperation level cannot invade. For $b > b^*$, the strategies with $P^C \leq P^{C^*}$ are evolutionarily stable. Here,

$$P^{C^*} = \frac{b}{b-1}\frac{1}{k+1}, \tag{3.17}$$

$$b^* = \frac{k+1}{k}, \tag{3.18}$$

where k is the degree of the regular graph. These values are obtained from the intersection point of two lines, $P^C_A = P^C_B$ and $F^k_A = F^{k-1}_B$. In the section bounded by these lines, F^k_A is larger than F^{k-1}_B and the fraction of B-type players is $2/N$, where N is the number of players. The group with $P^C > P^{C^*}$ can be destroyed and turned into the group with lower cooperation level, in the worst case, $P^C = 0$. Meanwhile, the group with $P^C \leq P^{C^*}$ can maintain its own cooperation level.

In our game rules, the group of more prudent players(the players under Rule

II) is more resistant to the invasion of strategy with low cooperation level. In reality, the informations on neighbors are likely to be limited, and are inaccurate sometimes. Hence, the players which collect the informations on all the neighbors to choose the fittest strategy aren't be expected to exist in the real world. However, if the players try to collect more information and determine the strategy more cautiously in the real-world PD game situations, it is likely that the propagation of strategies with low cooperation level can be blocked or at least retarded. According to the experiments for human subjects [81], approximately 70% of players imitate the strategies of the neighbors with the highest payoffs.

We introduced the mixed strategies into the spatial evolutionary PD games. As a bridge connecting the pure-strategy PD games and the mixed-strategy PD games, we studied the evolutionary stability. In the evolutionary stability study, we investigate two-strategy games similar to PD games with two pure strategies. The games with two mixed strategies are *de facto* PD games with two pure strategies. The game with two mixed strategies($P_A^C > P_B^C$) defined by a payoff matrix T, R, P, S is equivalent to the PD game with $T^{'}$, $R^{'}$, $P^{'}$, $S^{'}$. Here, the mixed strategy A and B are mapped to the pure strategy C and D respectively, and

$$\begin{aligned}
T^{'} &= P_B^C(R \cdot P_A^C + S(1-P_A^C)) + (1-P_B^C)(T \cdot P_A^C + P(1-P_A^C)) \\
R^{'} &= P_A^C(R \cdot P_A^C + S(1-P_A^C)) + (1-P_A^C)(T \cdot P_A^C + P(1-P_A^C)), \\
P^{'} &= P_B^C(R \cdot P_B^C + S(1-P_B^C)) + (1-P_B^C)(T \cdot P_B^C + P(1-P_B^C)), \\
S^{'} &= P_A^C(R \cdot P_B^C + S(1-P_B^C)) + (1-P_A^C)(T \cdot P_B^C + P(1-P_B^C)).
\end{aligned} \tag{3.19}$$

However, the difference between the pure-strategy game and the mixed-strategy game exists. In the pure-strategy games, we cannot compare the different strategies defined by different payoff matrixes. On the other hand, in the mixed-strategy games, we can compare the different strategies defined by the different P^C. Moreover, the games with more than three strategies are possible. In the real world, many players interact with the diverse strategies. Hence, to investigate the games in the real world, we need to study the games with various strategies including mixed strategies. Our study on the games with mixed strategies can be the basis for the studies on PD games with various strategies.

3.10 Summary

A mixed strategy is the strategy which mixes pure strategies with probabilities. It is one of the main concepts in game theory. It is known that all the finite games have the equilibrium points called as Nash equilibrium. This statement is valid only when players can select mixed strategies. Some games have Nash equilibrium which does not consist of only pure strategies.

In prisoner's dilemma games, a mixed strategy can be represented as the probability or level of cooperation. In the classical PD games, the optimal strategy is the strategy of defection. In the real world, however, the behaviors of players are observed, which can be interpreted as mixed strategies. Sometimes the most commonly observed strategies are mixed strategy.

So far, many previous studies on spatial PD games considered only two strategies, which are cooperation and defection. Our study introduces mixed strategies into the spatial PD games. As a first step, we studied the evolutionary stability in the spatial PD games with mixed strategies.

The evolutionary stability is a subject to investigate whether a stable state where a main strategy dominates can be maintained in a natural selection process. In this study, we investigate the propagation of a initially rare strategy in the system where all except one player have the same strategy. If the main strategy blocks the propagation of initially rare strategies, then the main strategy is evolutionarily stable.

The rule of games are as follows. An asynchronous updating rule is used. A candidate player which has the chance to change its strategy is selected randomly. The candidate chooses one of the neighbors as a reference. If the reference has higher fitness, then the candidate imitates the strategy of the reference. We use two rules for a candidate to choose a reference. In Rule I, a candidate chooses a reference randomly among one of the neighbors. In Rule II, a candidate selects the player which has the largest fitness among the neighbors as a reference. We compared the propagation behaviors under each rule.

The propagation behavior of a initially rare strategy is determined by the inequality of fitnesses. The inequality of fitnesses is determined by the cooperation probabilities of two strategies, the value of temptation, and the degree of a regular graph. In the rules we use, the same inequality of fitnesses leads to the same propagation behavior. For fixed temptation value and number of neighbors, the parameter space of the cooperation probabilities of two strategies can be divided into the sections for the same inequality of fitnesses.

When the newly introduced strategy has higher cooperation probability than the existing one, the new one cannot propagate on both rules. Meanwhile, when the cooperation probability of new strategy is lower than that of the existing one, the propagation behavior differs on each rule. Under Rule I, new strategy always propagates. New strategy propagates to all the players, or new strategy and old one coexist. Hence, the appearance of new strategy with lower cooperation probability leads to the fall of the cooperation level of the system. Under Rule I, the only evolutionarily stable strategy is the one with zero cooperation probability.

Under Rule II, in the parameter space of two cooperation probability, there always exist the sections where a new strategy cannot invade into the system even though the new one has lower cooperation probability than the existing one. Further, there always exist the evolutionarily stable strategies with non-zero cooperation probabilities; the strategy is ESS if it has the cooperation probability lower than the specific value related to the value of temptation and the number of neighbors. When the value of temptation is lower than the specific value, all the strategies are ESS; the system with the cooperation probability of 1 can maintain its own cooperation level.

In the PD games under Rule II, there exist the evolutionarily stable strategies, and the cooperation probability of system can be maintained. Under Rule I, the information that the candidate player needs is the fitness of one of the neighbors. Under Rule II, the player needs to know the fitnesses of all the neighbors, which takes more effort. If players try to collect more information to determine their strategies, the level of cooperation can be maintained, or at least the propagation speed of the strategy with lower cooperation probability can be slowed.

Chapter 4

Conclusion

Cooperation is a universal behavior observed in animal and human societies. Even though defection is the most beneficial choice to individuals, cooperation emerges. As a framework to study the origin of cooperation, prisoner's dilemma games are widely used. Here, prisoner's dilemma(PD) represents the situation where the individual optimum is defection while the global optimum is cooperation.

As the classical game theory which assumes the rationality of players cannot explain the emergence of cooperation in PD situation, many mechanisms to emerge cooperation have been proposed. One of them is evolutionary game, which assumes the natural selection process instead of the rationality of players, hence explains the emergence of cooperation among the players which lack rationality or information. A strategy is embedded in the gene or meme of a player. In the natural selection process, the players with inferior strategies vanish, and the number of the players with fitter strategies increases. It is found that in the evolutionary game on spatial structure, the clusters of cooperators are formed, hence the strategy of cooperation can survive. This thesis studies two variations of spatial evolutionary game. One is the variation of spatial structure, and the other is the variation of strategy.

Santos, *et al*. showed that in small-world scale-free networks, the fraction of cooperators is high. This is due to the degree heterogeneity and the close connection among hubs, which are the characteristics of small-world scale-free networks. In the family of scale-free networks, however, there exist the networks which have the rare connection among hubs. These networks are called as fractal or large-world networks. These networks have modular structure. Chapter 2 dealt with the PD games on fractal scale-free networks under the rules of Santos, *et al*. We focused on the formation of cooperator clusters. While one giant cluster of cooperators appears in a small-world scale-free network, a variety of clusters with various sizes emerges in a fractal scale-free network. As the lack of connection among hubs hinders the propagation of strategies among modules, the strategy of cooperation stays in modules. Many real networks are found to be fractal networks. If the players on the fractal networks behave according to the rules of Santos, *et al*., the addition of connection among hubs can enhance the level of cooperation.

Mixed strategy is an essential element in game theory. Yet, the spatial PD games with mixed strategies have been studied rarely. In PD games with two pure strategy of cooperation and defection, the mixed strategy can be represented by the cooperation probability, which shows the cooperation of a player. Chapter 3 introduced the spatial PD game with mixed strategies, and studied the evolutionary stability of mixed strategies. If the state that a specific strategy dominates a system can be maintained under the invasion of other strategies, the main strategy is evolutionarily stable. We compared the evolutionary stability under two rules. In Rule I, a candidate selects a reference among the neighbors randomly. In Rule II, a candidate chooses the neighbor with the highest fitness as reference. The player under Rule II needs more information than the one under Rule I. When the cooperation probability of the existing strategy is lower than that of new one, the new one cannot invade into the system regardless of the rules. When the existing strategy has higher cooperation probability than new one, the propagation behaviors differ by rules. Under Rule I, the new strategy always propagates globally, hence the only evolutionary stable strategy is the strategy with zero cooperation probability. The cooperation level of system cannot be maintained; the invasion of strategy with low cooperation probability lowers the cooperation level of system. Meanwhile, in the games under Rule II, in some cases, new strategy cannot propagate globally. Moreover, there exist the evolutionarily stable strategies with non-zero cooperation probability. When the temptation value is under a certain value, all the strategies are evolutionary stable. Hence, in the games under Rule II, the high cooperation level of system can be maintained. Although the players in the real world do not behave deterministically, if they try to collect more information and determine their actions prudentially, the cooperation level of system can be maintained, or at least decline slowly.

The evolutionary stability is related to the problems whether a rare, newly introduced strategy can propagate into the system globally. A meme, an idea and a life-style can be interpreted as strategies. Therefore, the propagation and replacement of a meme, an idea and a life-style can be studied in terms of evolutionary stability.

The introduction of mixed strategies can generalize the spatial PD games. The players can interact with various strategies. In the games with only pure strategies, the cooperation level of system is measured by the fraction of cooperators. In the games with mixed strategies, the distribution of cooperation probability of players determines the cooperation level of system. The distribution can differ by the structure of a network. The case which is known that all the players in system become defectors in the pure strategy games might be the case that cooperation exists with slight probability in the mixed strategy games.

We may rethink the results on chapter 2 in terms of mixed strategy. We concluded that to increase the number of cooperators, the connection among hubs should increase. In small-world networks where the connection among hubs are abundant, a giant cluster of cooperators is formed. The formation of a giant cluster surely raises the fraction of cooperators. However, it can be an Achilles' heel in the aspect of the maintenance of cooperation level. Assume the temptation value is high. If the strategy with high cooperation probability and that with slightly lower cooperation probability collide in the small-world scale-free network, it is likely that the strategy with higher cooperation probability becomes the major strategy. If a strategy with very low cooperation probability is introduced in this state, the giant cluster with high cooperation probability will vanish. Meanwhile, in a fractal scale-free network, the strategy with extremely low cooperation probability cannot propagate globally, or propagates slowly. Hence, in terms of the maintenance of cooperation level, fractal scale-free networks have advantages.

We might need to reexamine the previous results in PD games with pure strategies. The introduction of mixed strategies is not just a generalization of the games; it may be a paradigm shift in the spatial games.

Appendices

Appendix A

Propagation of strategies on cycle graph

In the game rules which we employ, the behaviors of propagation of a strategy are determined by the order of fitnesses. The cases in $b = 3$ cover all of the cases in $b = 1.5$ and $b = 2$, as shown in Table 9. Here, we focus on the results for $b = 3$.

A.1 Propagation of strategies under Rule I

In Rule I, a candidate player choose a reference player among its neighbors randomly. Shown in Fig. 28 is the fraction of players with initially rare startegy in each section.

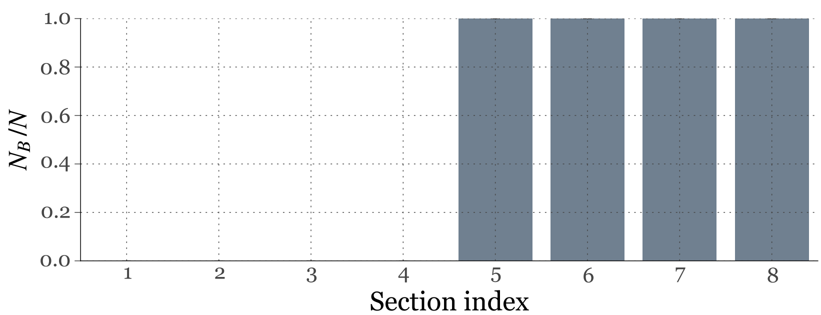


Figure 28: The fraction of B-type players in the mixed-strategy PD games under Rule I for b=3.0 on the cycle graph of $N = 1000$(Fig. 17(e)).

A.1.1 Section 1, 2, 3, 4 at $b = 3$

The propagation of strategies is depicted in Fig. 29. The color of a node represents the strategy; strategy A is black, and strategy B is blue. The size of a node represents

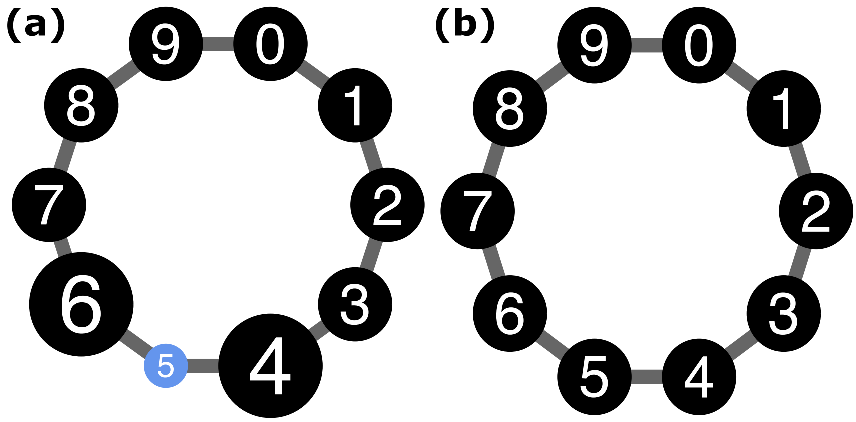


Figure 29: Propagation of strategies in section 1, 2, 3, 4 at $b = 3$ under Rule I on a cycle graph

the fitness of the node.

In section 1, 2, 3 and 4 at $b = 3$, F_B^2 is smaller than F_A^1. Consider player 5 is selected as a candidate player to try to change the strategy. The fitnesses of the neighbors are higher than that of player 5. Therefore, player 5 adopts the strategy of neighbors, strategy A. In the case of $P_A^C < P_B^C$, strategy B which is initially rare cannot survive.

A.1.2 Section 5, 6, 7, 8 at $b = 3$

In section 5, 6, 7 and 8 at $b = 3$, F_B^2 and F_B^2 are larger than F_A^1. The fitness of a B-type player is always higher than that of a A-type neighbor. Therefore, strategy B spreads to all the players, as shown in Fig. 30.

A.2 Propagation of strategies under Rule II

In Rule II, a candidate player chooses the neighbor with the highest fitness as a reference. Shown in Fig. 31 is the fraction of players with initially rare strategy in

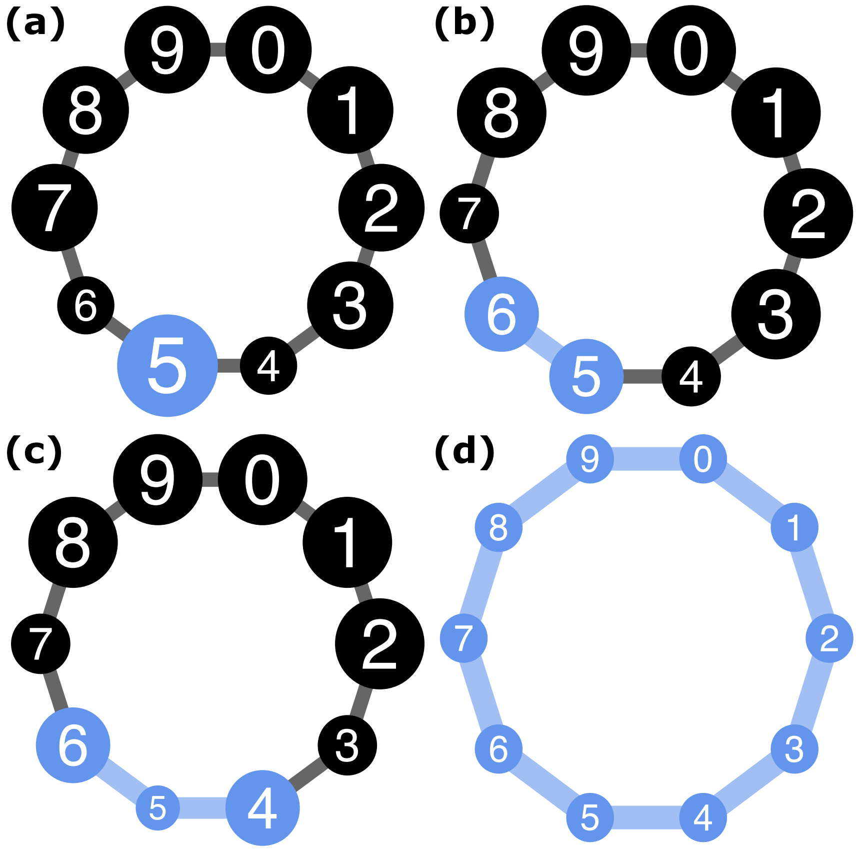


Figure 30: Propagation of strategies in section 5, 6, 7, 8 at $b = 3$ under Rule I on a cycle graph

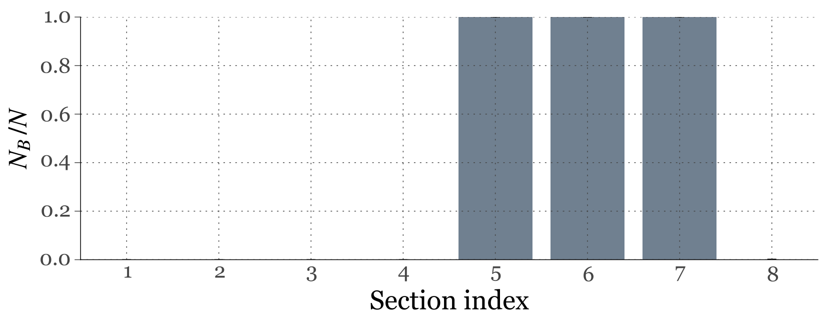


Figure 31: The fraction of B-type players in the mixed-strategy PD games under Rule II for b=3.0 on the cycle graph of $N = 1000$(Fig. 17(f)).

each section.

A.2.1 Section 1, 2, 3, 4 at $b = 3$

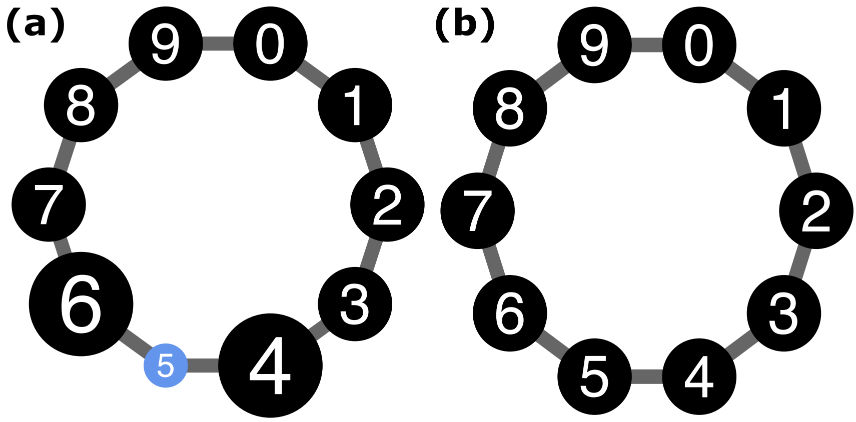


Figure 32: Propagation of strategies in section 1, 2, 3, 4 at $b = 3$ under Rule II on a cycle graph

In section 1, 2, 3 and 4 at $b = 3$, F_B^2 is smaller than F_A^1. The propagation of strategies is shown in Fig. 32. Consider player 5 is choosed as a candidate player to change the strategy. The fitnesses of the neighbors are higher than that of player 5. Therefore, player 5 adopts the strategy of neighbors, strategy A. In the case of $P_A^C < P_B^C$, strategy B which is initially rare cannot survive.

A.2.2 Section 5, 6, 7 at $b = 3$

In section 5, 6 and 7 at $b = 3$, F_B^2 and F_B^1 are larger than F_A^1 and F_A^2.

Consider a A-type player which has a B-type neighbor is selected as a candidate player to change the strategy. Among the neighbors of the A-type player, the B-type player has the highest fitness. Therefore, the A-type candidate adopts strategy B. As shown in Fig. 33, strategy B spreads to all the players.

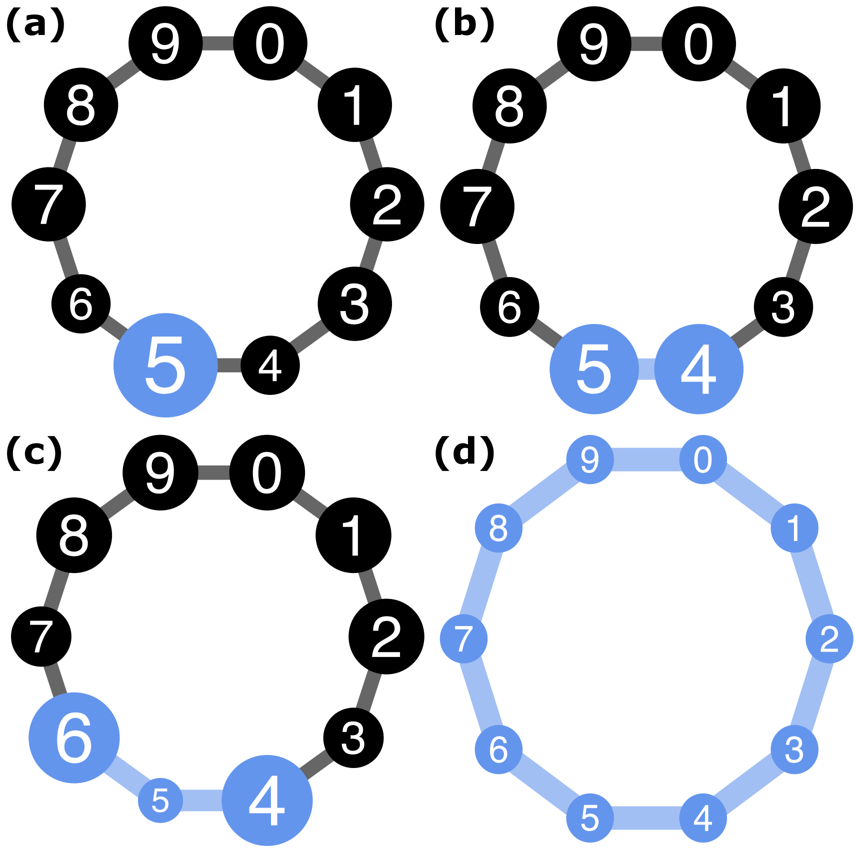


Figure 33: Propagation of strategies in section 5, 6, 7 at $b = 3$ under Rule II on a cycle graph

A.2.3 Section 8 at $b = 3$

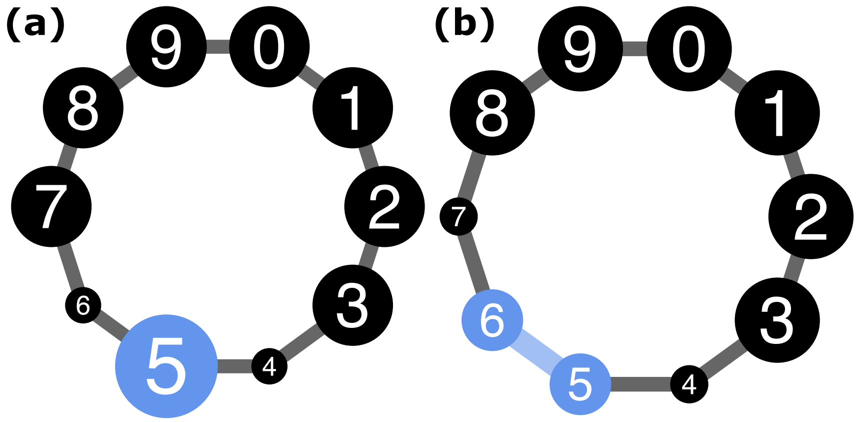


Figure 34: Propagation of strategies in section 8 at $b = 3$ under Rule II on a cycle graph

In section 8 at $b = 3$, F_B^2 and F_B^1 are larger than F_A^1. F_B^2 are larger than F_A^2. F_B^1 are smaller than F_A^2.

The propagation of strategies is shown in Fig. 34.

The strategy of a B-type player with two A-type neighbors spreads to the neighbor. However, the strategy of a B-type player with the neighbors of one A-type player and one B-type player doesn't spread. In Fig. 34(b), the neighbors of player 5, 6 with the highest fitness are player 6, 5. The neighbors of player 4, 7 with the highest fitness are player 3, 8, which have the same strategy with player 4, 7. Therefore, no players change their strategy. In this section, the final fraction of B-type players is $2/N$.

Appendix B

More detailed results on mixed-strategy PD games on honeycomb lattice

B.1 Propagation of strategies on honeycomb lattice under Rule I

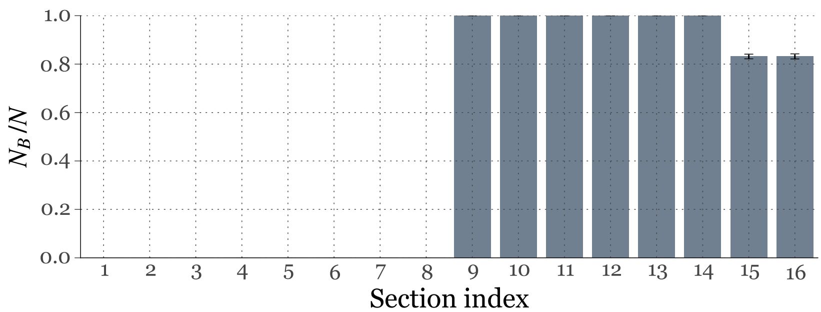


Figure 35: The fraction of B-type players in the mixed-strategy PD games by Rule I for b=3.0 on the honeycomb lattice of $N = 9800$(Fig. 21(e)).

The fraction of B-type players in the mixed-strategy PD games by Rule I for b=3.0 on the honeycomb lattice of $N = 9800$ are shown in Fig. 35. Data are averages over 200 configurations.

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, strategy B can invade. In section 9, 10, 11, 12, 13 and 14, all the players adopt strategy B, and strategy A vanishes.

In section 15 and 16, A-type players can survive, forming small clusters, as shown in Fig. 37 (a). As Fig. 37 (b) reveals, the fraction of B-type players appears irrelevant to the number of nodes in graph. In these sections, the following relation is satisfied commonly.

$$F_A^1 < F_B^1 < F_A^2 \tag{B.1}$$

B.2 Propagation of strategies on honeycomb lattice under Rule II

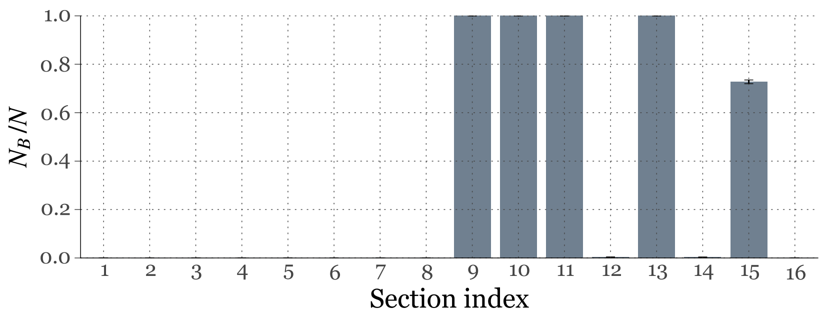


Figure 36: The fraction of B-type players in the mixed-strategy PD games by Rule II for b=3.0 on the honeycomb lattice of $N = 9800$(Fig. 21(f)).

The fraction of B-type players in the mixed-strategy PD games by Rule II for b=3.0 on the honeycomb lattice of $N = 9800$ are shown in Fig. 36. Data are averages over 200 configurations.

For $P_A^C < P_B^C$, no B-type player survives. For $P_A^C > P_B^C$, strategy B can invade. In section 9, 10, 11 and 13, all the players adopt strategy B.

In section 12 and 14, strategy B can invade into A-type players, however, with very small fraction. A example is shown in Fig. 38 (a). As shown in Fig. 38 (b), the number of B-type players appears to be constant regardless of network sizes. In these sections, the following relation is satisfied.

$$F_A^2 < F_B^1 < F_A^3 < F_B^2 \tag{B.2}$$

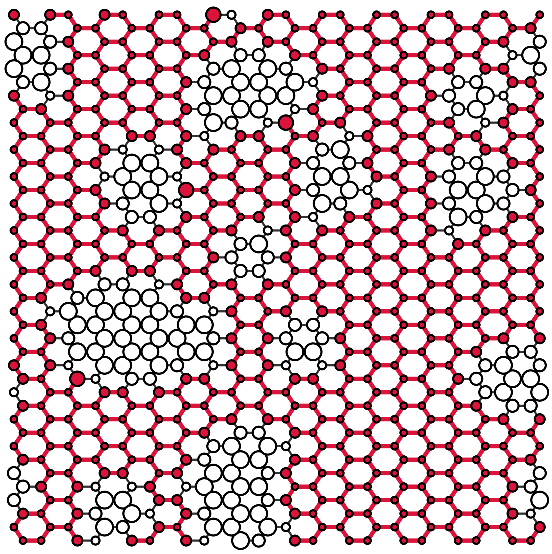
In section 15, strategy B prevails. Nonetheless, A-type players which form clusters can survive, as shown in Fig. 39 (a). The fraction of B-type players is approximately 0.73 regardless of network sizes(Fig. 39 (b)). In this section, the following relation is satisfied.

$$F_B^1 < F_A^2 < F_A^3 < F_B^2 \tag{B.3}$$

Section 16 is the section where only two B-player can invade(Fig. 40). In this section, the following relation is satisfied.

$$F_A^2 < F_B^2 < F_A^3 \tag{B.4}$$

(a)



(b)

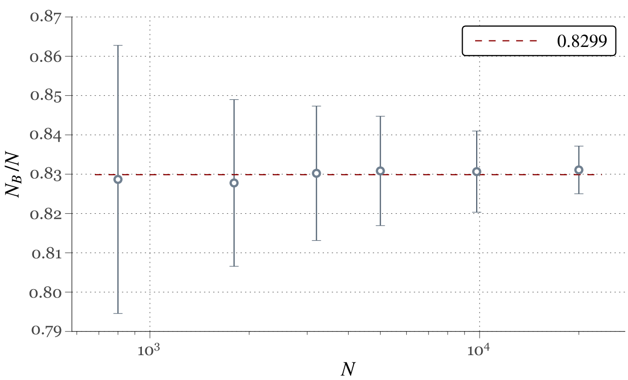
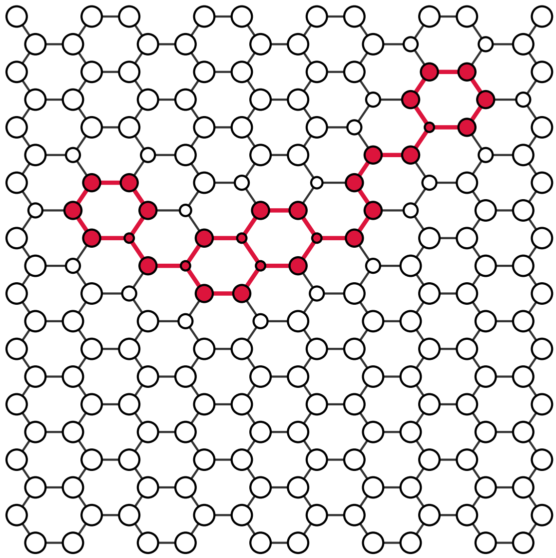


Figure 37: The propagation of strategies under Rule I for $b = 3.0$ on honeycomb lattice in section 15 and 16. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The fraction of B-type players against the number of players.

(a)



(b)

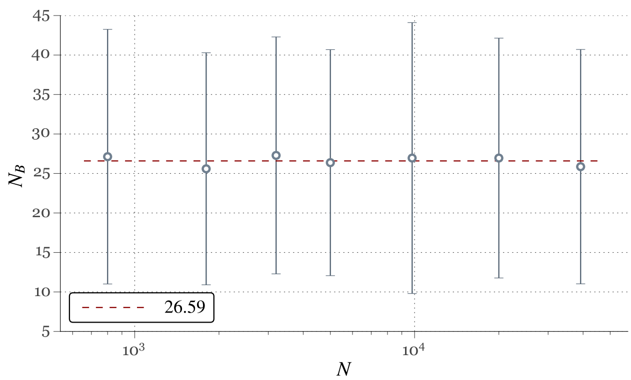
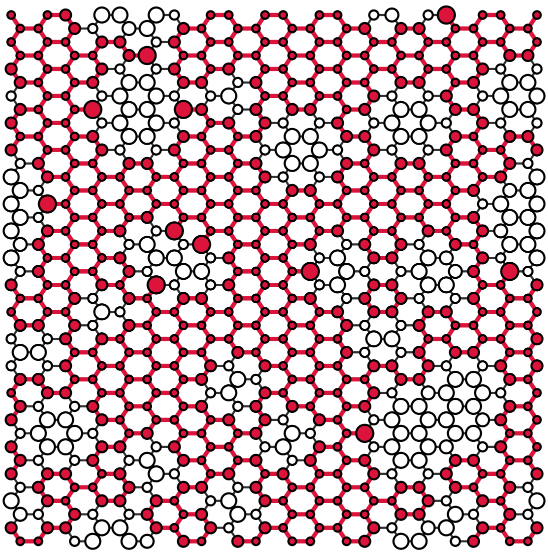


Figure 38: The propagation of strategies under Rule II for $b = 3.0$ on honeycomb lattice in section 12 and 14. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The number of B-type players against the number of players.

(a)



(b)

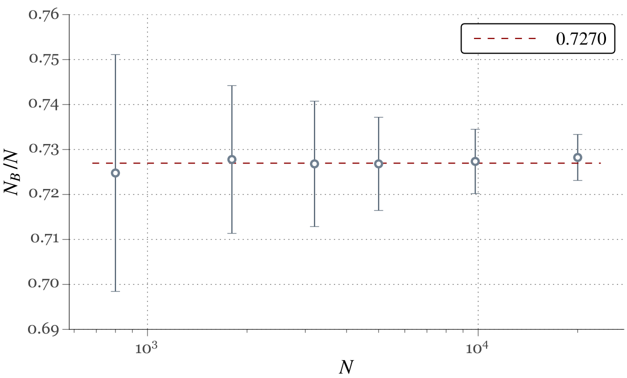


Figure 39: The propagation of strategies under Rule II for $b = 3.0$ on honeycomb lattice in section 15. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The fraction of B-type players against the number of players.

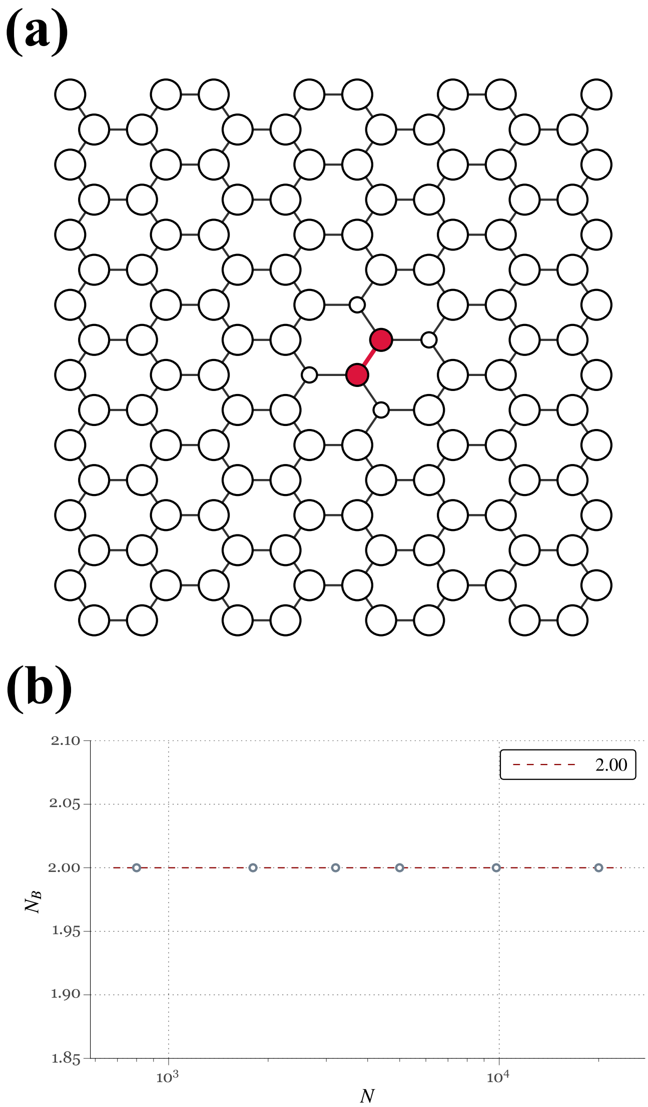


Figure 40: The propagation of strategies under Rule II for $b = 3.0$ on honeycomb lattice in section 16. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The number of B-type players against the number of players.

Appendix C

More detailed results on mixed-strategy PD games on square lattice

C.1 Propagation of strategies on square lattice under Rule I

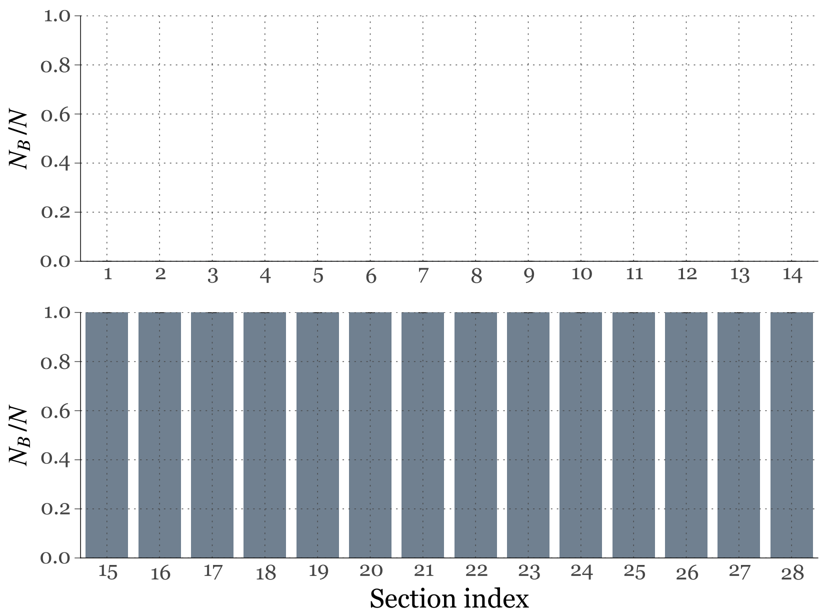


Figure 41: The fraction of B-type players in the mixed-strategy PD games under Rule I for b=3.0 on the square lattice of $N = 10000$(Fig. 26(e)).

The fraction of B-type players in the mixed-strategy PD games by Rule I for b=3.0 on the square lattice of $N = 10000$ are shown in Fig. 41. Data are averages over 200 configurations.

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, the fraction of B-type players is 1.

C.2 Propagation of strategies on square lattice under Rule II

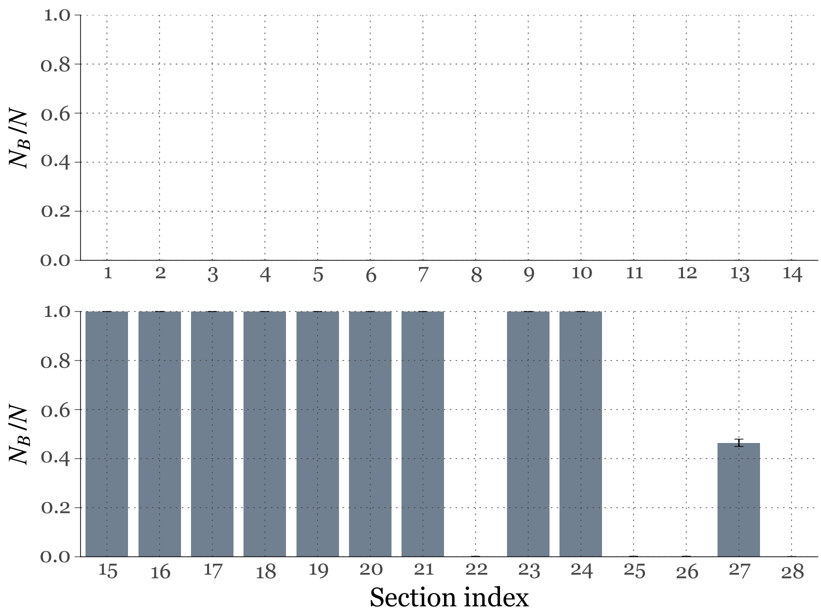


Figure 42: The fraction of B-type players in the mixed-strategy PD games under Rule II for b=3.0 on the square lattice of $N = 10000$(Fig. 26(f)).

The fraction of B-type players in the mixed-strategy PD games by Rule II for b=3.0 on the square lattice of $N = 10000$ are shown in Fig. 42. Data are averages over 200 configurations.

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, strategy B can invade. In section 15, 16, 17, 18, 19, 20, 21, 23 and 24, the fraction of B-type player is 1.

In section 22, 25 and 26, very small fraction of B-type player can invade(Fig. 43).

In these sections, the following relation is satisfied.

$$F_A^3 < F_B^2 < F_A^4 < F_B^3 \tag{C.1}$$

The number of B-type players is irrelevant to network sizes.

In section 27, the clusters of A-type players with diverse sizes are formed, as shown in Fig. 44 (a). The fraction of B-type players is about a half(Fig. 44 (b)). In this section, the following relation is satisfied.

$$F_B^2 < F_A^3 < F_A^4 < F_B^3 \tag{C.2}$$

In section 28, the number of B-type players is 2(Fig. 45). In this section, the following relation is satisfied.

$$F_A^3 < F_B^3 < F_A^4 \tag{C.3}$$

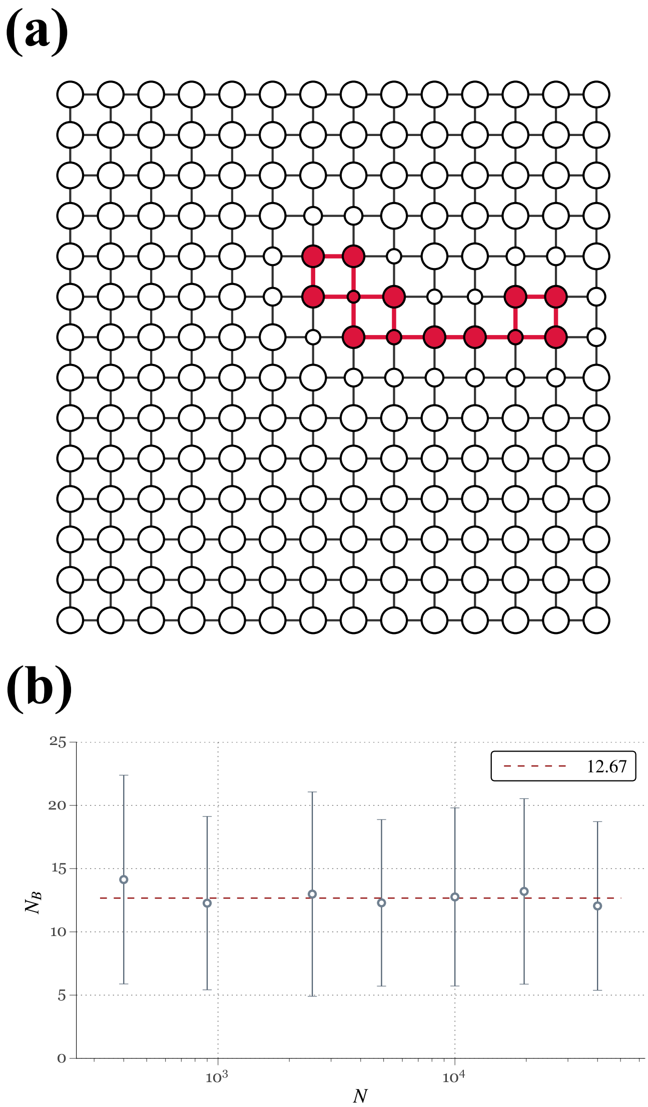
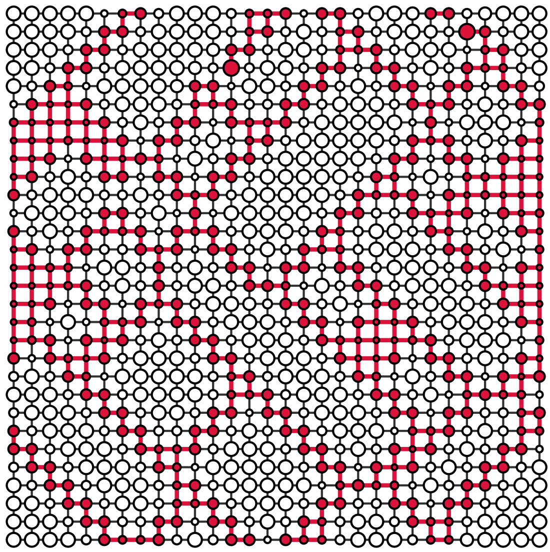


Figure 43: The propagation of strategies under Rule II for $b = 3.0$ on square lattice in section 22, 25 and 26. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The number of B-type players against the number of players.

(a)



(b)

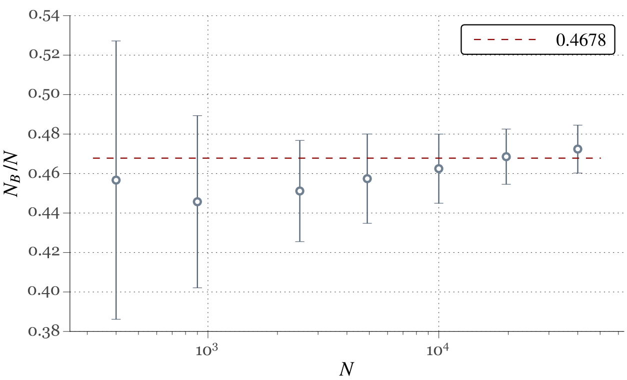
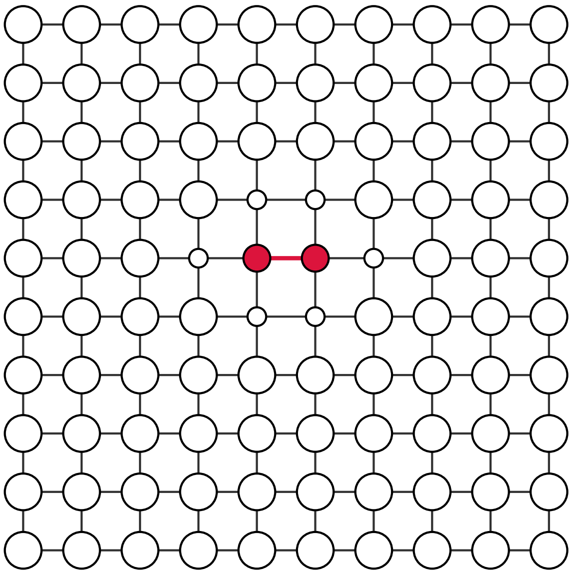


Figure 44: The propagation of strategies under Rule II for $b = 3.0$ on square lattice in section 27. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The fraction of B-type players against the number of players.

(a)



(b)

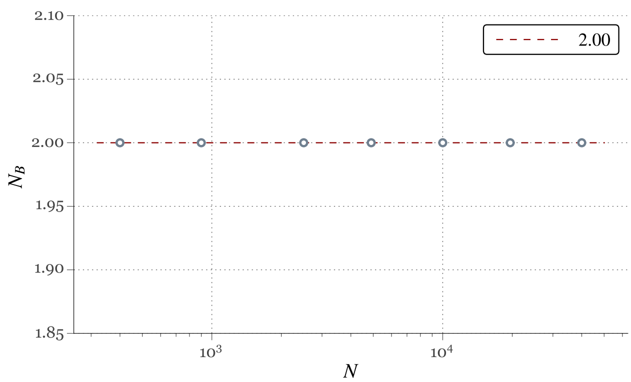


Figure 45: The propagation of strategies under Rule II for $b = 3.0$ on square lattice in section 28. (a) a snapshot of a stable state. The white(red) circles are A-type(B-type) players and the size of a circle represents the fitness of a player. (b) The number of B-type players against the number of players.

Appendix D

More detailed results on mixed-strategy PD games on random graphs with degree 3

D.1 Size dependency of fraction of *B*-type players on random regular graphs with degree 3 under Rule I

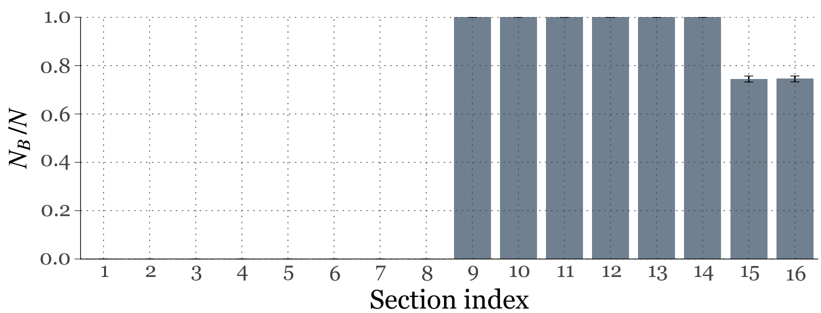


Figure 46: The fraction of B-type players in the mixed-strategy PD games under Rule I for b=3.0 on the random regular graphs of $N = 10000$ with degree 3(Fig. 22(e)).

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, strategy B can invade. In section 9, 10, 11, 12, 13 and 14, the fraction of B-type players is 1.

In section 15 and 16, the fraction of B-type players is approximately 0.75 regardless of network sizes.

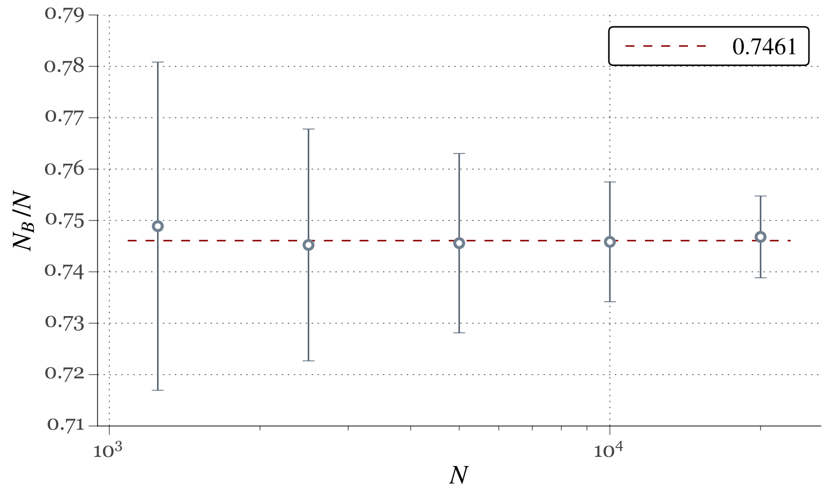


Figure 47: The fraction of B-type players against the number of players in the PD game under Rule I for $b = 3.0$ on random regular graphs with degree 3 in section 15 and 16.

D.2 Size dependency of fraction of B-type players on random regular graphs with degree 3 under Rule II

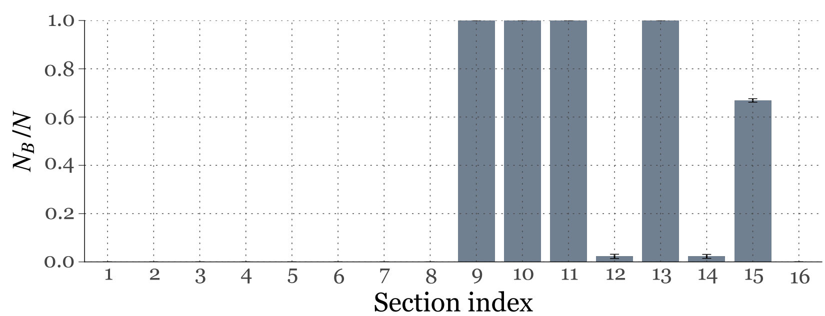


Figure 48: The fraction of B-type players in the mixed-strategy PD games under Rule II for b=3.0 on the random regular graphs of $N = 10000$ with degree 3(Fig. 22(f)).

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, at least two B-

type players can survive. In section 9, 10, 11 and 13, the fraction of B-type players is 1.

In section 12 and 14, the fraction of B-type players is approximately proportional to $N^{-0.5}$, as shown in Fig. 49 (a). As N goes to infinity, the fraction of B-type players goes to 0.

In section 15, as Fig. 49 (b) reveals, the fraction of B-type players is about 0.67, irrelevant to network sizes.

It is indicated from Fig. 49 (c) that in section 16, the number of B-type players is 2.

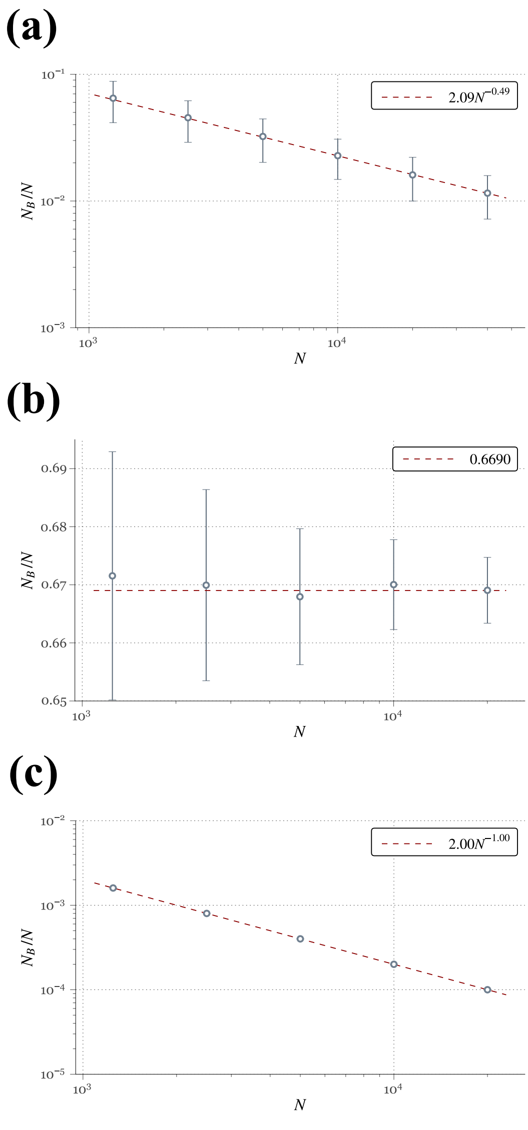


Figure 49: The fraction of B-type players against the number of players in the PD game under Rule II for $b = 3.0$ on random regular graphs with degree 3. (a) Section 12 and 14, (b) section 15, and (c) section 16.

Appendix E

More detailed results on mixed-strategy PD games on random graphs with degree 4

E.1 Size dependency of fraction of B-type players on random regular graphs with degree 4 under Rule I

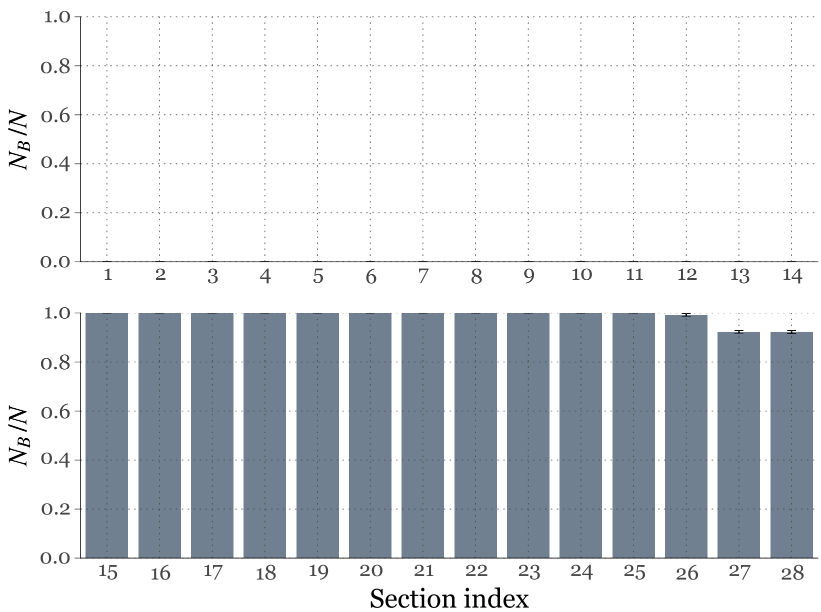


Figure 50: The fraction of B-type players in the mixed-strategy PD games under Rule I for b=3.0 on the random regular graphs of $N = 10000$ with degree 4(Fig. 27(e)).

For $P_A^C < P_B^C$, strategy B vanishes. For $P_A^C > P_B^C$, strategy B can survive. In section 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, the fraction of B-type players is 1.

In section 26, the fraction of B-type players is about 0.99, as shown in Fig. 51 (a). Fig. 51 (b) reveals that In section 27 and 28, the fraction of B-type players is about 0.92.

E.2 Size dependency of fraction of *B*-type players on random regular graphs with degree 4 under Rule II

For $P_A^C < P_B^C$, the fraction of B-type players is 0. For $P_A^C > P_B^C$, strategy B can invade. In section 15, 16, 17, 19, 21, 22, 24, 25, the fraction of B-type players is 1.

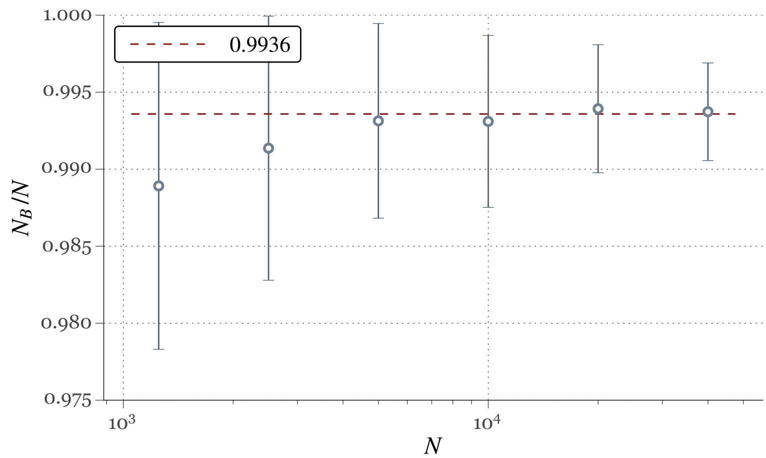
In section 18, 20 and 23, the fraction of B-type players is about 0.995, as shown in Fig. 53 (a). Fig. 53 (b) reveals that in section 21 and 24, the fraction of B-type players is about 0.95.

It is indicated from Fig. 54 (a) that in section 22, 25 and 26, the fraction of B-type players is proportional to $N^{-0.5}$. As N goes to infinity, the fraction of B-type players goes to 0.

Fig. 54 (b) shows that in section 27, the fraction of B-type players is about 0.52, regardless of network sizes.

It is seen from Fig. 54 (c) that in section 28, the number of B-type players is 2.

(a)



(b)

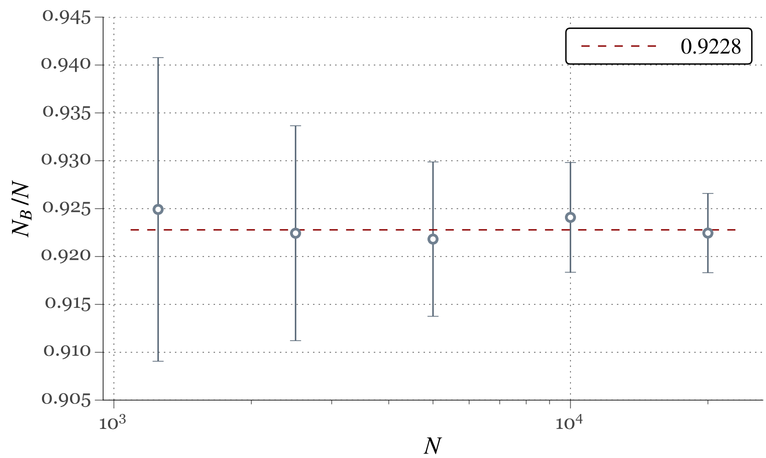


Figure 51: The fraction of B-type players against the number of players in the PD game under Rule I for $b = 3.0$ on random regular graphs with degree 4. (a) Section 26, (b) section 27 and 28.

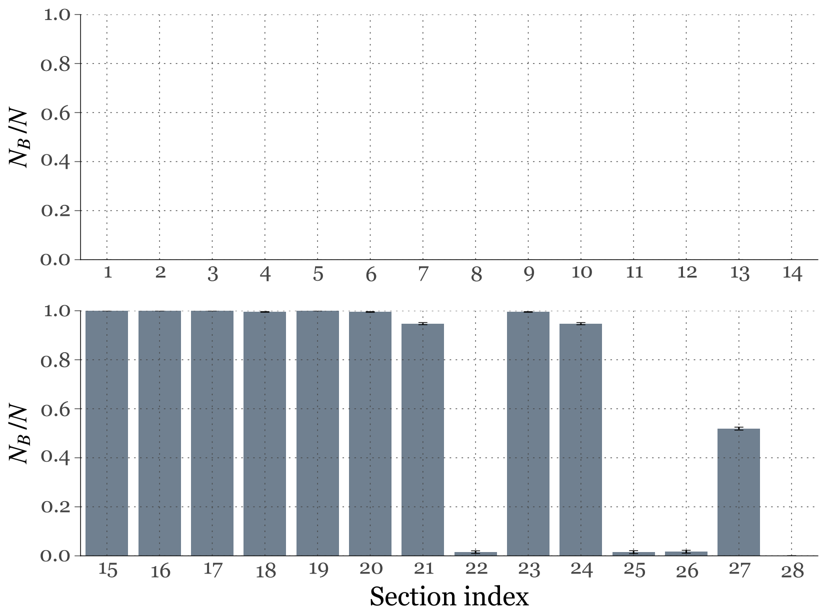


Figure 52: The fraction of B-type players in the mixed-strategy PD games under Rule II for b=3.0 on the random regular graphs of $N = 10000$ with degree 4(Fig. 27(f)).

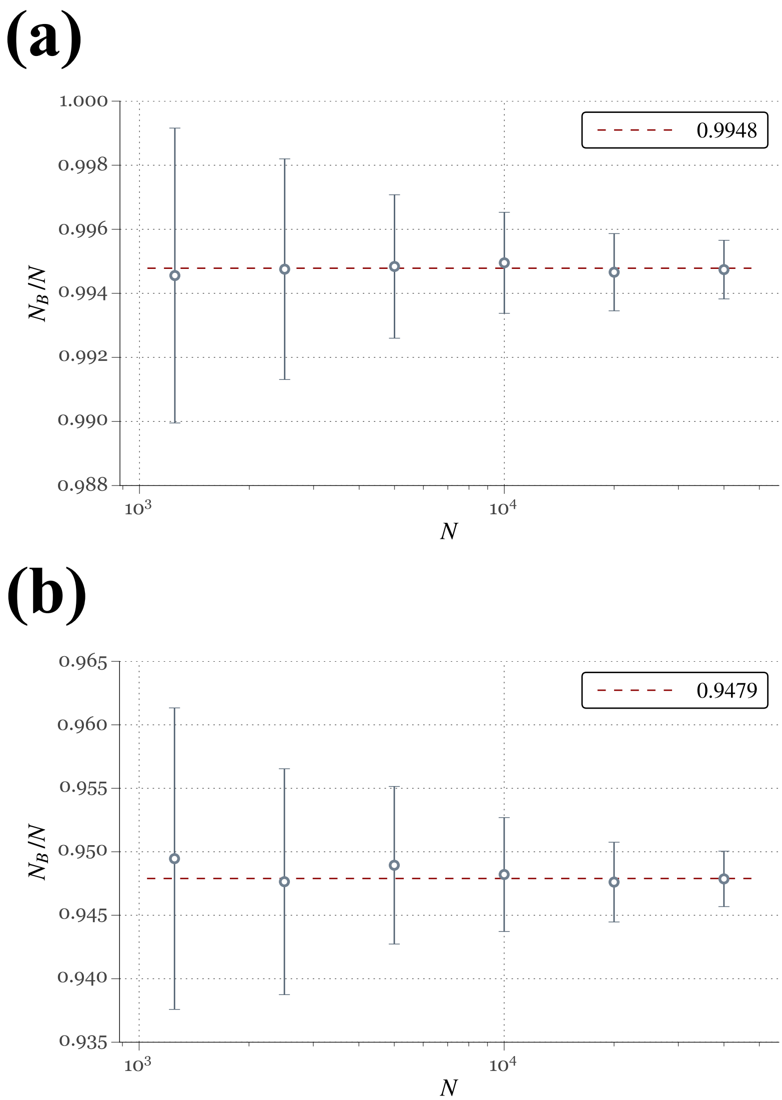


Figure 53: The fraction of B-type players against the number of players in the PD game under Rule II for $b = 3.0$ on random regular graphs with degree 4. (a) Section 18, 20 and 23, (b) section 21 and 24.

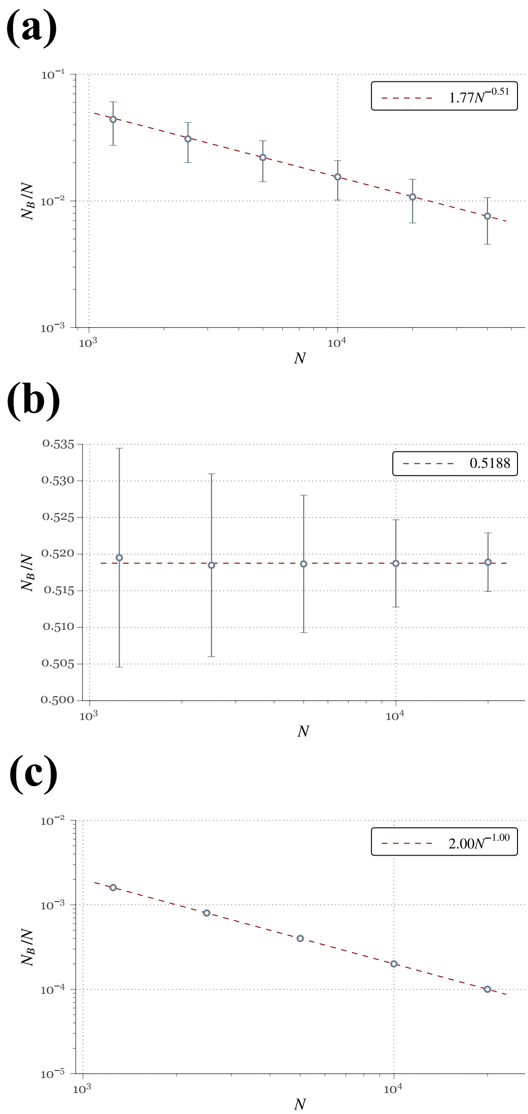


Figure 54: The fraction of B-type players against the number of players in the PD game under Rule II for $b = 3.0$ on random regular graphs with degree 4. (a) Section 22, 25 and 26, (b) section 27, (c) section 28.

Bibliography

[1] J. M. Smith, *Evolution and the Theory of Games* (Cambridge University Press, 1982).

[2] P. E. Turner and L. Chao, Nature **398**, 441 (1999).

[3] W. Hamilton, The American Naturalist **97**, 354 (1963).

[4] D. S. Wilson and E. Sober, Behavioral and Brain Sciences **17**, 585 (1994).

[5] R. Axelrod and W. Hamilton, Science **211**, 1390 (1981).

[6] R. L. Trivers, Quarterly review of biology , 35 (1971).

[7] M. A. Nowak and K. Sigmund, Nature **393**, 573 (1998).

[8] M. A. Nowak and K. Sigmund, Nature **437**, 1291 (2005).

[9] T. H. Clutton-Brock and G. A. Parker, Nature **373**, 209 (1995).

[10] E. Fehr and S. Gächter, Nature **415**, 137 (2002).

[11] M. Milinski, D. Semmann, and H. J. Krambeck, Nature **415**, 424 (2002).

[12] C. Hauert, S. De Monte, J. Hofbauer, and K. Sigmund, Science **296**, 1129 (2002).

[13] G. Szabó and C. Hauert, Phys. Rev. Lett. **89**, 118101 (2002).

[14] Z. Xu, Z. Wang, and L. Zhang, Phys Rev E **80**, 061104 (2009).

[15] M. A. Nowak, Science **314**, 1560 (2006).

[16] M. A. Nowak and R. May, Nature **359**, 826 (1992).

[17] M. A. Nowak, S. Bonhoeffer, and R. May, International Journal of Bifurcation and Chaos in Applied Sciences and Engineering **4**, 33 (1994).

[18] R. Jiménez, H. Lugo, J. A. Cuesta, and A. Sánchez, arXiv preprint arXiv:0706.0648 (2007).

[19] J. A. Cuesta, R. Jiménez, H. Lugo, and A. Sánchez, arXiv **q-bio.PE** (2007).

[20] Z.-X. Wu, X.-J. Xu, Z.-G. Huang, S.-J. Wang, and Y.-H. Wang, Phys Rev E **74**, 021107 (2006).

[21] F. Fu, C. Hauert, M. Nowak, and L. Wang, Phys Rev E **78**, 026117 (2008).

[22] S. Van Segbroeck, F. Santos, T. Lenaerts, and J. Pacheco, Phys. Rev. Lett. **102**, 058105 (2009).

[23] X. Chen, F. Fu, and L. Wang, Phys Rev E **80**, 051104 (2009).

[24] D.-M. Shi, H.-X. Yang, M.-B. Hu, W.-B. Du, B.-H. Wang, and X.-B. Cao, Physica A **388**, 4646 (2009).

[25] Z. Wang and M. Perc, Phys Rev E **82** (2010).

[26] M. Perc and Z. Wang, PLoS ONE **5**, e15117 (2010).

[27] A. Szolnoki and G. Szabó, EPL **77**, 30004 (2007).

[28] A. Szolnoki and M. Perc, New J. Phys. **10**, 043036 (2008).

[29] G. Szabó, A. Szolnoki, and J. Vukov, EPL **87**, 18007 (2009).

[30] A. Szolnoki, J. Vukov, and G. Szabó, Phys Rev E **80**, 056112 (2009).

[31] F. Fu, T. Wu, and L. Wang, Phys Rev E **79**, 036101 (2009).

[32] M. Perc and A. Szolnoki, Phys Rev E **77**, 011904 (2008).

[33] Z.-X. Wu and Y.-H. Wang, Phys Rev E **75**, 041114 (2007).

[34] M. Zhang and J. Yang, Phys Rev E **79**, 011121 (2009).

[35] X. Chen, F. Fu, and L. Wang, Phys Rev E **78**, 051120 (2008).

[36] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).

[37] R. Albert and A.-L. Barabási, Rev Mod Phys **74**, 47 (2002).

[38] F. Santos and J. Pacheco, Phys. Rev. Lett. **95** (2005).

[39] F. C. Santos, J. F. Rodrigues, and J. M. Pacheco, Proceedings of the Royal Society B: Biological Sciences **273**, 51 (2006).

[40] F. C. Santos, J. M. Pacheco, and T. Lenaerts, Proceedings of the National Academy of Sciences **103**, 3490 (2006).

[41] S. Devlin and T. Treloar, Phys Rev E **79**, 016107 (2009).

[42] S. Assenza, J. Gómez-Gardeñes, and V. Latora, Phys Rev E **78**, 017101 (2008).

[43] A. Pusch, S. Weber, and M. Porto, Phys Rev E **77**, 036120 (2008).

[44] X. Chen, F. Fu, and L. Wang, Physica A **378**, 512 (2007).

[45] Z. X. Wu, X. J. Xu, and Y. H. Wang, arXiv preprint physics/0508220 (2005).

[46] Z.-X. Wu, J.-Y. Guan, X.-J. Xu, and Y.-H. Wang, Physica A: Statistical Mechanics and its Applications **379**, 672 (2007).

[47] M. Tomassini, E. Pestelacci, and L. Luthi, International Journal of Modern Physics C **18**, 1173 (2007).

[48] A. Szolnoki, M. Perc, and Z. Danku, Physica A **387**, 2075.

[49] N. Masuda, Proceedings of the Royal Society B: Biological Sciences **274**, 1815 (2007).

[50] C. Hauert and G. Szabó, Am. J. Phys. **73**, 405 (2005).

[51] C. L. Tang, W. X. Wang, X. Wu, and B. H. Wang, The European Physical Journal B - Condensed Matter and Complex Systems **53**, 411 (2006).

[52] L. L. Jiang, M. Zhao, H. X. Yang, J. Wakeling, B. H. Wang, and T. Zhou, Phys Rev E **80**, 031144 (2009).

[53] B. A. Huberman and N. S. Glance, Proceedings of the National Academy of Sciences **90**, 7716 (1993).

[54] A. Mukherji, V. Rajan, and J. R. Slagle, Nature **379**, 125 (1996).

[55] H. Ohtsuki, C. Hauert, E. Lieberman, and M. A. Nowak, Nature **441**, 502 (2006).

[56] H. Ohtsuki and M. A. Nowak, Proceedings of the Royal Society B: Biological Sciences **273**, 2249 (2006).

[57] M. A. Nowak and R. M. May, International Journal of Bifurcation and Chaos in Applied Sciences and Engineering **3**, 35 (1993).

[58] M. A. Nowak, S. Bonhoeffer, and R. May, Proceedings of the National Academy of Sciences **91**, 4877 (1994).

[59] G. Szabó and C. Tőke, Phys Rev E **58**, 69 (1998).

[60] C. Hauert and M. Doebeli, Nature **428**, 643 (2004).

[61] E. Ravasz, A. L. Somera, D. A. Mongru, Z. N. Oltvai, and A.-L. Barabási, Science **297**, 1551 (2002).

[62] C. Song, S. Havlin, and H. A. Makse, Nature **433**, 392 (2005).

[63] K.-I. Goh, G. Salvi, B. Kahng, and D. Kim, Phys. Rev. Lett. **96** (2006).

[64] D. Lee, K.-I. Goh, B. Kahng, and D. Kim, Phys Rev E **82**, 026112 (2010).

[65] M. Hinczewski and A. N. Berker, Phys Rev E **73**, 066126 (2006).

[66] C.-K. Yun, N. Masuda, and B. Kahng, Phys Rev E **83**, 057102 (2011).

[67] A. N. Berker and S. Ostlund, Journal of Physics C: Solid State Physics **12**, 4961 (1979).

[68] A.-L. Barabási, R. Albert, and H. Jeong, Nature **401**, 130 (1999).

[69] A. Arenas, L. Danon, A. Díaz-Guilera, P. M. Gleiser, and R. Guimerà, The European Physical Journal B - Condensed Matter and Complex Systems **38**, 373 (2004).

[70] M. E. J. Newman, Phys Rev E **69**, 066133 (2004).

[71] G. Palla, I. Derényi, I. Farkas, and T. Vicsek, Nature **435**, 814 (2005).

[72] Y.-Y. Ahn, J. P. Bagrow, and S. Lehmann, Nature **466**, 761 (2010).

[73] U. Raghavan, R. Albert, and S. Kumara, Phys Rev E **76**, 036106 (2007).

[74] A. Lancichinetti, S. Fortunato, and J. Kertész, New J. Phys. **11**, 033015 (2009).

[75] J. F. Nash, Proceedings of the National Academy of Sciences **36**, 48 (1950).

[76] J. Nash, The Annals of Mathematics **54**, 286 (1951).

[77] J. M. Smith and G. R. Price, Nature (1973).

[78] W. G. S. Hines, Theoretical Population Biology **31**, 195 (1987).

[79] H. Ohtsuki and M. A. Nowak, Journal of Theoretical Biology **251**, 698 (2008).

[80] F. Schweitzer, L. Behera, and H. Mühlenbein, Adv Complex Syst **05**, 269 (2013).

[81] A. Traulsen, D. Semmann, R. D. Sommerfeld, H. J. Krambeck, and M. Milinski, Proceedings of the National Academy of Sciences **107**, 2962 (2010).

초 록

다양한 분야에서 협력이 일어나는 기제를 이해하기 위한 도구로 죄수의 딜레마 게임을 사용해왔다. 수많은 연구에서 협력을 설명하기 위한 다양한 가설들이 제시되었다. 성공적이라 평가되는 가설 중 하나는 진화 과정과 공간 구조의 조합이다. 첫 번째 장에서 공간 구조 위의 진화적 죄수의 딜레마 게임을 간단히 살펴보겠다. 그 다음 두 파트에서 두 가지 세부적인 측면에서 공간 구조 위의 진화적 죄수의 딜레마 게임을 연구한 결과를 제시하였다.

두 번째 장은 큰 세상 네트워크에서 작은 세상 네트워크로 변화가 가능한 네트워크 위에서 진행되는 죄수의 딜레마 게임에 대한 연구이다. 이 연구에서는 특히 협력 전략을 유지하는 행위자들이 이루는 집단에 대해 살펴보았다. 허브들 간의 연결이 많은 작은 세상 네트워크에서는 단 하나의 협력자 집단이 생성되며 전체적인 협력 수준도 높다. 반면, 큰 세상 네트워크에서는 다양한 크기를 갖는 수많은 협력자 집단이 형성되며, 협력자 비율은 상대적으로 높지 않다. 큰 세상 네트워크에서 작은 세상 네트워크로 네트워크를 변화시키면서 협력자 집단의 크기 분포를 조사하였고, 전이점에서는 크기 분포가 멱함수 꼴을 따른다는 점을 확인하였다.

세 번째 장에서는 진화적 죄수의 딜레마 게임에 혼합 전략을 도입했다. 죄수의 딜레마 게임에서 혼합 전략은 행위자의 협력 확률로써 표현 가능하다. 적용 사례로서, 레귤러 그래프 위에서 두 가지 혼합 전략만으로 진행되는 죄수의 딜레마 게임에서 진화적 안정성을 조사했다. 다른 전략의 침입을 허용하지 않는 전략을 진화적으로 안정한 전략이라고 한다. 결정론적인 게임 법칙 하에서는 항상 진화적으로 안정한 전략이 존재한다는 점을 확인했다. 이러한 전략을 가진 집단은 다른 전략의 침입 시도에도 불구하고 본래의 협력 수준을 유지할 수 있다. 죄수의 딜레마 게임에 혼합 전략을 도입한 이 연구는 보다 현실에 가까운

게임의 기초가 될 수 있을 것이다.

주요어 : 죄수의 딜레마 게임, 프랙털 네트워크, 큰세상 네트워크, 작은세상 네트워크, 혼합전략, 진화적 안정성

학번 : 2004-20413